## Chapter 4 Instructor Notes

The chapter starts by developing the dynamic equations for energy storage elements. The analogy between electrical and hydraulic circuits (*Make The Connection: Fluid (hydraulic) Capacitance*, p. 138, *Make The Connection: Fluid (hydraulic)Iinertance*, p. 150, Table 4.2) is introduced early to permit a connection with ideas that may already be familiar to the student from a course in fluid mechanics, such as mechanical, civil, chemical and aerospace engineers are likely to have already encountered. A *Focus on Measurements* boxes*: Capacitive displacement transducer and microphones*, pp. 147-148, permits approaching the subject of capacitance in a pragmatic fashion, if so desired. The instructor wishing to gain a more in-depth understanding of such transducers will find a detailed analysis in[1].

Next, signal sources are introduced, with special emphasis on sinusoids. The material in this section can also accompany a laboratory experiment on signal sources. The emphasis placed on sinusoidal signals is motivated by the desire to justify the concepts of phasors and impedance, which are introduced next. The author has found that presenting the impedance concept early on is an efficient way of using the (invariably too short) semester or quarter. The chapter is designed to permit a straightforward extension of the resistive circuit analysis concepts developed in Chapter 3 to the case of dynamic circuits excited by sinusoids. The ideas of nodal and mesh analysis, and of equivalent circuits, can thus be reinforced at this stage. The treatment of AC circuit analysis methods is reinforced by the usual examples and drill exercises, designed to avoid unnecessarily complicated circuits. Two *Focus on Methodology* boxes (pp. 165 and 180) provide the student with a systematic approach to the solution of basic AC analysis problems using phasor and impedance concepts.

The capacitive displacement transducer example is picked up again in *Focus on Measurements: Capacitive displacement transducer* (pp.175-177) to illustrate the use of impedances in a bridge circuit. This type of circuit is very common in mechanical measurements, and is likely to be encountered at some later time by some of the students.

The homework problems in this chapter are mostly exercises aimed at mastery of the techniques.

**Learning Objectives**

1. Compute currents, voltages and energy stored in capacitors and inductors.
2. Calculate the average and root-mean-square value of an arbitrary (periodic) signal.
3. Write the differential equation(s) for circuits containing inductors and capacitors.
4. Convert time-domain sinusoidal voltages and currents to phasor notation, and vice-versa, and represent circuits using impedances.
5. Apply the circuit analysis methods of Chapter 3 to AC circuits in phasor form.

[1] E. O. Doebelin, Measurement Systems – Application and Design, 4th Edition, McGraw-Hill, New York, 1990.

# Section 4.1: Energy Storage Elements

## Problem 4.1

### Solution:

**Known quantities:**
Inductance value, $L = 0.5$ H ; the current through the inductor as a function of time.

**Find:**
The voltage across the inductor, (Eq. 4.9), as a function of time.

**Assumptions:**
$i_L(t \le 0) = 0$

**Analysis:**
Using the differential relationship for the inductor, we may obtain the voltage by differentiating the current:

$$v_L(t) = L\frac{di_L(t)}{dt} = 0.5\frac{di_L(t)}{dt} = 0.5 \times \left[-377 \times 2\, sin\left(377t + \frac{\pi}{6}\right)\right]$$

$$= 377\, sin\left(377t + \frac{\pi}{6} - \pi\right) = 377\, sin\left(377t - \frac{5\pi}{6}\right) \text{ V}$$

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## Problem 4.2

### Solution:

**Known quantities:**
Capacitance value $C = 100$ μF ; capacitor terminal voltage as a function of time.

**Find:**
The current through the capacitor as a function of time for each case:

a) $v_c(t) = 40\cos(20t - \pi/2)V$

b) $v_c(t) = 20\sin(100t)V$

c) $v_c(t) = -60\sin(80t + \pi/6)V$

d) $v_c(t) = 30\cos(100t + \pi/4)V$ .

**Assumptions:**
The capacitor is initially discharged: $v_C(t = 0) = 0$

**Analysis:**
Using the defining differential relationship for the capacitor, (Eq. 4.4), we may obtain the current by differentiating the voltage:

$$i_C(t) = C\frac{dv_C(t)}{dt} = 100 \times 10^{-6}\frac{dv_C(t)}{dt} = 10^{-4}\frac{dv_C(t)}{dt}$$

a)

$$i_C(t) = 10^{-4}\left[-20\times 40\,sin\left(20t - \frac{\pi}{2}\right)\right] = -0.08\,sin\left(20t - \frac{\pi}{2}\right)$$
$$= 0.08\,sin\left(20t - \frac{\pi}{2} + \pi\right) = 0.08\,sin\left(20t + \frac{\pi}{2}\right) \text{ A}$$

b)

$$i_C(t) = 10^{-4}\left[100\times 20\,cos 100t\right] = 0.2\,cos 100t \text{ A}$$

c)

$$i_C(t) = 10^{-4}\left[-80\times 60\,cos\left(80t + \frac{\pi}{6}\right)\right] = -0.48\,cos\left(80t + \frac{\pi}{6}\right)$$
$$= 0.48\,cos\left(80t + \frac{\pi}{6} - \pi\right) = 0.48\,cos\left(80t - \frac{5\pi}{6}\right) \text{ A}$$

d)

$$i_C(t) = 10^{-4}\left[-100\times 30\,sin\left(100t + \frac{\pi}{4}\right)\right] = -0.3\,sin\left(100t + \frac{\pi}{4}\right)$$
$$= 0.3\,sin\left(100t + \frac{\pi}{4} - \pi\right) = 0.3\,sin\left(100t - \frac{3\pi}{4}\right) \text{ A}$$

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## Problem 4.3

### *Solution:*

**Known quantities:**
Inductance value, $L = 250 \text{ mH}$; the current through the inductor, as a function of time.

**Find:**
The voltage across the inductor as a function of time for each case

a) $i_L(t) = 5\sin 25tA$

b) $i_L(t) = -10\cos 50tA$

c) $i_L(t) = 25\cos(100t + \pi/3)A$

d) $i_L(t) = 20\sin(10t - \pi/12)A$.

**Assumptions:**
$i_L(t \le 0) = 0$

**Analysis:**
Using the differential relationship for the inductor, (Eq. 4.9), we may obtain the voltage by differentiating the current:

$$v_L(t) = L\frac{di_L(t)}{dt} = 250\times 10^{-3}\frac{di_L(t)}{dt} = 0.25\frac{di_L(t)}{dt}$$

a)

$$v_L(t) = 0.25\left[25\times 5\,cos\,25t\right] = 31.25\,cos\,25t \text{ V}$$

b)

$$v_L(t) = 0.25\left[-50\times(-10\,sin\,50t)\right] = 125\,sin\,50t \text{ V}$$

c)

$$v_L(t) = 0.25\left[-100\times 25\, sin\left(100t+\frac{\pi}{3}\right)\right] = -625\, sin\left(100t+\frac{\pi}{3}\right)$$

$$= 625\, sin\left(100t+\frac{\pi}{3}-\pi\right) = 625\, sin\left(100t-\frac{2\pi}{3}\right) \text{ V}$$

d)

$$v_L(t) = 0.25\left[10\times 20\, cos\left(10t-\frac{\pi}{12}\right)\right] = 50\, cos\left(10t-\frac{\pi}{12}\right) \text{ V}$$

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## Problem 4.4

### *Solution:*

**Known quantities:**
Inductance value; resistance value; the current through the circuit shown in Figure P4.4 as a function of time.

**Find:**
The energy stored in the inductor as a function of time.

**Analysis:**
The magnetic energy stored in an inductor may be found from, (Eq. 4.16):

$$w_L(t) = \frac{1}{2}Li(t)^2 = \frac{1}{2}(2)i^2(t) = i^2(t)$$

For $-\infty < t < 0$,

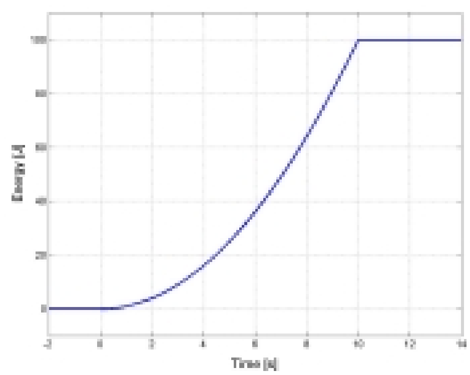
$$w_L(t) = 0$$

For $0 \le t < 10\,s$

$$w_L(t) = t^2 \text{ J}$$

For $10\ s \le t < +\infty$

$$w_L(t) = 100 \text{ J}$$

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## Problem 4.5

### *Solution:*

**Known quantities:**
Inductance value; resistance value; the current through the circuit in Figure P4.4 as a function of time.

**Find:**
The energy delivered by the source as a function of time.

**Analysis:**
The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the inductor:

$$w_S(t) = w_R(t) + w_L(t) = Ri^2(t) + \frac{1}{2}Li^2(t)$$

$$= (1)i^2(t) + \frac{1}{2}(2)i^2(t) = 2i^2(t)$$

For $-\infty < t < 0$,

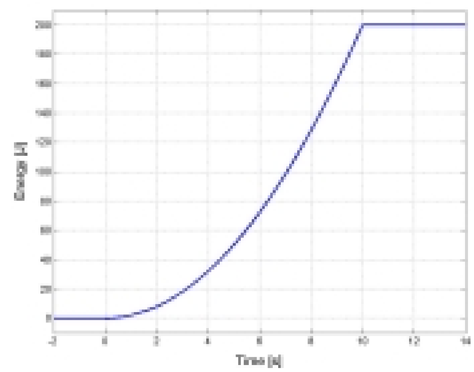
$$w_S(t) = 0$$

For $0 \le t < 10\,s$

$$w_S(t) = 2t^2 \text{ J}$$

For $10\ s \le t < +\infty$

$$w_S(t) = 200 \text{ J}$$



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## Problem 4.6

### *Solution:*

**Known quantities:**
Inductance value; resistance value; the current through the circuit shown in Figure P4.4 as a function of time.

**Find:**
The energy stored in the inductor and the energy delivered by the source as a function of time.

**Analysis:**
The magnetic energy stored in an inductor may be found from, (Eq. 4.16):

$$w_L(t) = \frac{1}{2}Li(t)^2 = \frac{1}{2}(2)i^2(t) = i^2(t)$$

For $-\infty < t < 0$,

$$w_L(t) = 0$$

For $0 \le t < 10\,s$

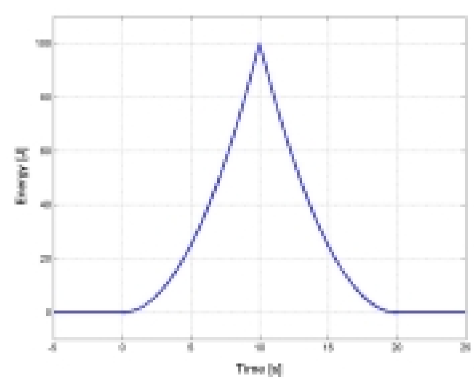
$$w_L(t) = t^2 \text{ J}$$

For $10 \le t < 20\,s$

$$w_L(t) = (20-t)^2 = 400 - 40t + t^2 \text{ J}$$

For $20\ s \le t < +\infty$

$$w_L(t) = 0 \text{ J}$$



The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the inductor:

$$w_S(t) = w_R(t) + w_L(t) = Ri^2(t) + \frac{1}{2}Li^2(t)$$

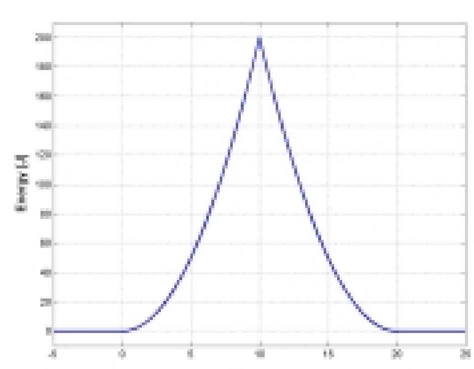
$$= (1)i^2(t) + \frac{1}{2}(2)i^2(t) = 2i^2(t)$$

For $-\infty < t < 0$,

$$w_L(t) = 0$$

For $0 \le t < 10\,s$

$$w_L(t) = 2t^2 \text{ J}$$



For $10 \le t < 20\,s$

$$w_L(t) = 2\,(20-t)^2 = 800 - 80t + 2t^2 \text{ J}$$

For $20\ s \le t < +\infty$

$$w_L(t) = 0 \text{ J}$$

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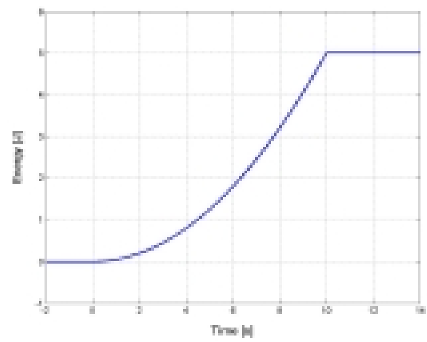
## Problem 4.7

### *Solution:*

**Known quantities:**
Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

**Find:**
The energy stored in the capacitor as a function of time.

**Analysis:**
The energy stored in a capacitor may be found from:



$$w_C(t) = \frac{1}{2}Cv(t)^2 = \frac{1}{2}(0.1)v(t)^2 = 0.05v(t)^2$$

For $-\infty < t < 0$

$$w_C(t) = 0$$

For $0 \le t < 10s$

$$w_C(t) = 0.05\,t^2 \text{ J}$$

For $10s \le t < +\infty$

$$w_C(t) = 0.05(10)^2 = 5 \text{ J}$$

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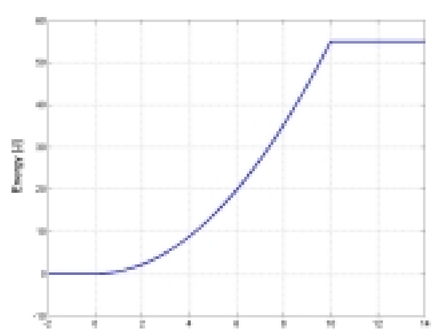
## Problem 4.8

### *Solution:*

**Known quantities:**
Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

**Find:**
The energy delivered by the source as a function of time.

**Analysis:**
The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the capacitor:



$$w_S(t) = w_R(t) + w_C(t) = \frac{v^2(t)}{R} + \frac{1}{2}Cv^2(t)$$

$$= \frac{1}{2}v^2(t) + \frac{1}{2}(0.1)v^2(t) = 0.55i^2(t)$$

For $-\infty < t < 0$,

$$w_S(t) = 0$$

For $0 \le t < 10\,s$

$$w_S(t) = 0.55t^2 \text{ J}$$

For $10\ s \le t < +\infty$

$$w_S(t) = 55 \text{ J}$$

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## Problem 4.9

### *Solution:*

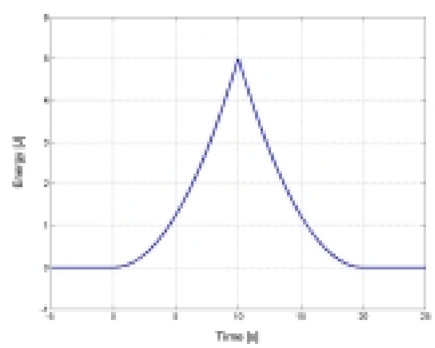
### *Solution:*

**Known quantities:**
Capacitance value; resistance value; the voltage applied to the circuit shown in Figure P4.7 as a function of time.

**Find:**
The energy stored in the capacitor and the energy delivered by the source as a function of time.

**Analysis:**
The energy stored in a capacitor may be found from:

$$w_C(t) = \frac{1}{2}Cv(t)^2 = \frac{1}{2}(0.1)v(t)^2 = 0.05v(t)^2$$



For $-\infty < t < 0$

$$w_C(t) = 0$$

For $0 \le t < 10s$

$$w_C(t) = 0.05\,t^2 \text{ J}$$

For $10 \le t < 20s$

$$w_C(t) = 0.05\,(20-t)^2 = 20 - 2t + 0.05t^2 \text{ J}$$

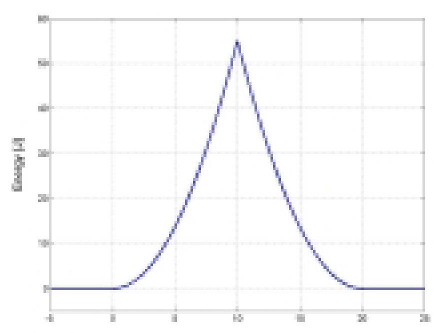
For $10s \le t < +\infty$

$$w_C(t) = 0$$

The energy delivered by the source is the sum of the energy absorbed by the resistance and the energy stored in the capacitor:

$$w_S(t) = w_R(t) + w_C(t) = \frac{v^2(t)}{R} + \frac{1}{2}Cv^2(t)$$

$$= \frac{1}{2}v^2(t) + \frac{1}{2}(0.1)v^2(t) = 0.55i^2(t)$$



For $-\infty < t < 0$

$$w_C(t) = 0$$

For $0 \le t < 10s$

$$w_C(t) = 0.55\,t^2 \text{ J}$$

For $10 \le t < 20s$

$$w_C(t) = 0.55\,(20-t)^2 = 220 - 22t + 0.55t^2 \text{ J}$$

For $10s \le t < +\infty$

$$w_C(t) = 0$$

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## Problem 4.10

### *Solution:*

**Known quantities:**

Capacitance, resistance and inductance values; the source voltage $v_S = 6\ V$ applied to the circuit shown in Figure P4.10.

**Find:**

The energy stored in each capacitor and inductor.

**Analysis:**

Under steady-state conditions, all the currents are constant, no current can flow through the capacitors, and the voltage across any inductor is equal to zero.

$$v_{2F} = v_{4\Omega} \Rightarrow \frac{6 - v_{4\Omega}}{2} = \frac{v_{4\Omega}}{4} + \frac{v_{4\Omega}}{8} \Rightarrow v_{4\Omega} = 3.43\ \mathbf{V}$$

$$\Rightarrow w_{2F} = \frac{1}{2} C_{2F} v_{2F}^2 = \frac{1}{2}(2\ \mathbf{F})(3.43\ \mathbf{V})^2 = 11.76\ \mathbf{J}$$

$$v_{1F} = v_{2H} = 0 \qquad \Rightarrow w_{1F} = \frac{1}{2} C_{1F} v_{1F}^2 = \frac{1}{2}(1\ F)(0)^2 = 0$$

$$i_{2H} = \frac{v_{4\Omega}}{8} = 0.43\ \mathbf{A} \quad \Rightarrow w_{2H} = \frac{1}{2} L_{2H} i_{2H}^2 = \frac{1}{2}(2\ \mathbf{H})(0.43\ \mathbf{A})^2 = 0.18\ \mathbf{J}$$

$$v_{3F} = v_{4\Omega} = 3.43\ \mathbf{V} \quad \Rightarrow w_{3F} = \frac{1}{2} C_{3F} v_{3F}^2 = \frac{1}{2}(3\ \mathbf{F})(3.43\ \mathbf{V})^2 = 17.65\ \mathbf{J}$$

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## Problem 4.11

### *Solution:*

**Known quantities:**

Capacitance, resistance and inductance values; the voltage $v_A = 12\ \text{V}$ applied to the circuit shown in Figure P4.11.

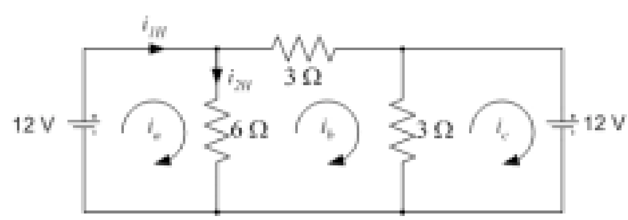
**Find:**

The energy stored in each capacitor and inductor.

**Analysis:**

Under steady-state conditions, all the currents are constant, no current can flow across the capacitors, and the voltage across any inductor is equal to zero.

The voltage for the 1-F capacitor is equal to the 12-Volt input.




Since the voltage is the same on either end of the 3-Ω resistor in parallel with the 2-F capacitor, there is no voltage drop through either component.

Finally, since there is no voltage drop through the 3-Ω resistor in parallel with the 2-F capacitor, there is no current flow through the resistor, and the current through the 1-H inductor is equal to the current through the 2-H inductor.

Therefore,

$$v_{1F} = v_A = 12 \text{ V} \Rightarrow w_{1F} = \frac{1}{2} C_{1F} v_{1F}^2 = \frac{1}{2}(1 \text{ F})(12 \text{ V})^2 = 72 \text{ J}$$

$$i_{1H} = \frac{12\text{V}}{6\Omega} = 2 \text{ A} \quad \Rightarrow w_{1H} = \frac{1}{2} L_{1H} i_{1H}^2 = \frac{1}{2}(1 \text{ H})(2 \text{ A})^2 = 2 \text{ J}$$

$$i_{2H} = i_{1H} = 2 \text{ A} \Rightarrow w_{2H} = \frac{1}{2} L_{2H} i_{2H}^2 = \frac{1}{2}(2 \text{ H})(2 \text{ A})^2 = 4 \text{ J}$$

$$v_{2F} = 0 \text{ V} \Rightarrow w_{2F} = \frac{1}{2} C_{2F} v_{2F}^2 = \frac{1}{2}(2 \text{ F})(0 \text{ V})^2 = 0 \text{ J}$$

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## Problem 4.12

### *Solution:*

**Known quantities:**
Capacitance value $C = 80\ \mu\text{F}$ ; the voltage applied to the capacitor as a function of time as shown in Figure P4.12.

**Find:**
The current through the capacitor as a function of time.

**Analysis:**
Since the voltage waveform is piecewise continuous, the derivative must be evaluated over each continuous segment.

For $0 < t < 5$ ms

$$v_C(t) = m_{v_C} t + q_{v_C}$$

where:

$$m_{v_C} = \frac{[-10 \text{ V}] - [+20 \text{ V}]}{[5 \text{ ms}] - [0]} = -6 \ \frac{\text{V}}{\text{ms}}$$

$$q_{v_C} = +20 \text{ V}$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left[m_{v_C} t + q_{v_C}\right] = Cm_{v_C} = (80\ \mu\text{F})\left(-6\ \frac{\text{V}}{\text{ms}}\right) = -480 \text{ mA}$$

For $5 \text{ ms} < t < 10 \text{ ms}$

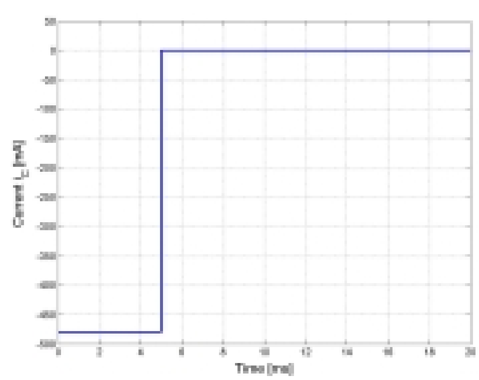
$$v_C(t) = -10 \text{ V}$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[-10 \text{ V}] = 0$$

For $t > 10$ ms

$$v_C(t) = 0$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[0] = 0$$



A capacitor is fabricated from two conducting plates separated by a dielectric constant. Dielectrics are also isolators; therefore, current cannot really flow through a capacitor. Positive charge, however, entering one plate exerts a repulsive force on and forces positive carriers to exit the other plate. Current then *appears* to flow through the capacitor. Such currents are called *electric displacement currents*.

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## Problem 4.13

### *Solution:*

**Known quantities:**
Inductance value, $L = 35 \text{ mH}$; the voltage applied to the inductor as shown in Figure P4.12; the initial condition for the current $i_L(0) = 0$.

**Find:**
The current across the inductor as a function of time.

**Analysis:**
Since the voltage waveform is piecewise continuous, integration must be performed over each continuous segment. Where not indicated *t* is supposed to be expressed in seconds.

For $0 < t \le 5 \text{ ms}$

$$v_L(t) = m_{v_L} t + q_{v_L}$$

where:

$$m_{v_L} = \frac{[-10 \text{ V}] - [+20 \text{ V}]}{[5 \text{ ms}] - [0]} = -6 \ \frac{\text{V}}{\text{ms}}$$

$$q_{v_L} = +20 \text{ V}$$

$$i_L = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)d\tau = 0 + \frac{1}{L}\int_0^t \left(m_{v_L}\tau + q_{v_L}\right)d\tau = \frac{1}{L}\left[\frac{1}{2}m_{v_L}\tau^2 + q_{v_L}\tau\right]_0^t =$$

$$= \frac{1}{2L}\left(m_{v_L}t^2 + q_{v_L}t\right) = \frac{1}{2\cdot 35 \text{ mH}}\left(-6 \ \frac{\text{V}}{\text{ms}}\cdot t^2 + 20 \text{ V}\cdot t\right) = \left(-85.71\cdot 10^3 t^2 + 571.4t\right) \text{ A}$$

$$i_L(t = 5 \text{ ms} = 0.005 \text{ s}) = \left(-85.71\cdot 10^3 \cdot (0.005)^2 + 571.4\cdot 0.005\right) \text{ A} = 714.3 \text{ mA}$$

For $5 \text{ ms} < t \le 10 \text{ ms}$

$$v_L(t) = c_{v_L} = -10 \text{ V}$$

$$i_L = i_L(0.005) + \frac{1}{L}\int_{0.005}^t v_L(\tau)d\tau = i_L(0.005) + \frac{1}{L}\int_{0.005}^t \left(c_{v_L}\right)d\tau = i_L(0.005) + \frac{1}{L}\left[c_{v_L}\tau\right]_{0.005}^t =$$

$$= 714.3 \text{ mA} - \frac{1}{35 \text{ mH}}\cdot(-10 \text{ V})\cdot(t - 0.005 \text{ s}) = (2.143 - 285.7t) \text{ A}$$

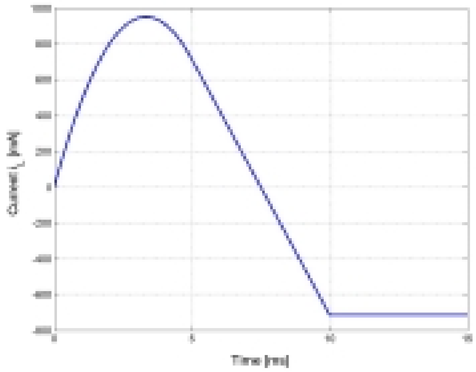
$$i_L(t = 10 \text{ ms} = 0.01 \text{ s}) = (2.143 - 285.7(0.01)) \text{ A} = -713.5 \text{ mA}$$

For $t > 10 \text{ ms}$

$$v_L(t) = c_{v_L} = 0$$

$$i_L = i_L(0.01) + \frac{1}{L}\int_{0.01}^t v_L(\tau)d\tau = i_L(0.01) + \frac{1}{L}\int_{0.01}^t (0)d\tau = i_L(0.01) + \frac{1}{L}[0]_{0.005}^t =$$

$$= i_L(0.01) = -713.5 \text{ mA}$$



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### Problem 4.14

## *Solution:*

**Known quantities:**

Inductance value $L = 0.75 \text{ mH}$; the voltage applied to the inductor as a function of time as shown in Figure P4.14.

**Find:**

The current through the inductor at the time $t = 15 \text{ µs}$ .

**Assumptions:**

$$i_L(t \le 0) = 0$$

**Analysis:**

Since the voltage waveform is a piecewise continuous function of time, integration must be performed over each continuous segment. Where not indicated, *t* is expressed in seconds.

For $0 < t \le 5 \text{ µs}$

$$v_L(t) = m_{v_L} t + q_{v_L}$$

where:

$$m_{v_L} = \frac{[3.5 \text{ V}] - [0 \text{ V}]}{[5 \text{ µs}] - [0]} = 0.7 \ \frac{\text{V}}{\text{µs}}$$

$$q_{v_L} = 0 \text{ V}$$

For $t > 5 \text{ µs}$

$$v_L(t) = c_{v_L} = -1.9 \text{ V}$$

Therefore:

$$i_L(t = 15 \text{ µs}) = \frac{1}{L}\int_{-\infty}^{15 \text{ µs}} v_L(\tau) d\tau = i_L(0) + \frac{1}{L}\int_0^{5 \text{ µs}} (m_{v_L}\tau) d\tau + \frac{1}{L}\int_{5 \text{ µs}}^{15 \text{ µs}} (c_{v_L}) d\tau =$$

$$= i_L(0) + \frac{1}{L}\left[\frac{1}{2} m_{v_L} \tau^2\right]_0^{5 \text{ µs}} + \frac{1}{L}\left[c_{v_L}\tau\right]_{5 \text{ µs}}^{15 \text{ µs}} = 0 + \frac{1}{0.75 \textbf{ mH}} \cdot \frac{0.7 \ \frac{\textbf{V}}{\textbf{ms}}}{2} \cdot \left((5 \text{ µs})^2 - 0\right) +$$

$$+ \frac{1}{0.75 \textbf{ mH}} \cdot (-1.9 \textbf{ V}) \cdot (15 \text{ µs} - 5 \text{ µs}) = -13.67 \textbf{ mA}$$

---

## Problem 4.15

### *Solution:*

**Known quantities:**
Capacitance value $C = 680 \text{ nF}$; the periodic voltage applied to the capacitor as shown in Figure P4.15:
$v_{Peak} = 20 \text{ V}$, $T = 40 \text{ µs}$ .

**Find:**
The waveform and the plot for the current through the capacitor as a function of time.

**Analysis:**
Since the voltage waveform is not a continuous function of time, differentiation can be performed only over each continuous segment. In the discontinuity points the derivative of the voltage will assume an infinite value, the sign depending on the sign of the step. Where not indicated, *t* is expressed in seconds.

For each period $0 < t < T,\ T < t < 2T,\ \ldots$ the behavior of the capacitor will be the same; thus, we consider only the first period:
For $0 < t < T$

$$v_C(t) = m_{v_C} t + q_{v_C}$$

where:

$$m_{v_C} = \frac{[v_{Peak}] - [0]}{[T] - [0]} = 0.5 \ \frac{\text{V}}{\text{µs}}$$

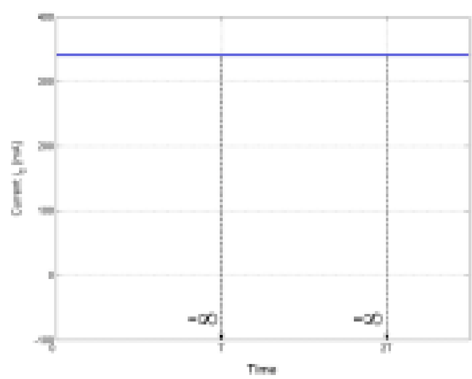
$$q_{v_C} = 0 \text{ V}$$

$$i_C = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}\left[m_{v_C} t\right] = Cm_{v_C} = \left(480 \ \frac{\text{nAs}}{\text{V}}\right)\left(0.5 \ \frac{\text{V}}{\text{µs}}\right) = 340 \text{ mA}$$

For $t = T,\ 2T,\ \ldots$

$$i_C = C\frac{dv_C(t)}{dt} = C[-\infty]$$

Figure P 4.15 shows the current waveform.
Note: For the voltage across the capacitor to decrease instantaneously to zero at $t = T,\ 2T,\ \ldots$, the charge on the plates of the capacitor should be instantaneously discharged. This requires an infinite current which is not physically possible.
If this were a practical waveform, the slope at $t = T,\ 2T,\ \ldots$, would be finite, not infinite. A large negative spike of current over a finite period of time would result instead of the infinite spike over zero time. These large spike of current (or voltage) degrade the performance of many circuits.



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## Problem 4.16

### *Solution:*

**Known quantities:**
Inductance value, $L = 16 \text{ µH}$ ; the voltage applied to the inductor as a function of time as shown in Figure P4.16; the initial condition for the current $i_L(0) = 0$ .

**Find:**
The current through the inductor at $t = 30\ \mu s$ .

**Analysis:**
Since the voltage waveform is piecewise continuous, the integration can be performed over each continuous segment. Where not indicated *t* is supposed to be expressed in seconds.

$$i_L(t = 30\ \mu s) = \frac{1}{L}\int_{-\infty}^{30\ \mu s} v_L(\tau)d\tau = i_L(0) + \frac{1}{L}\int_0^{20\ \mu s} v_L(\tau)d\tau + \frac{1}{L}\int_{20\ \mu s}^{30\ \mu s} v_L(\tau)d\tau =$$

$$= i_L(0) + \frac{1}{L}\left[\frac{3}{3}\tau^3\ \frac{V}{s^2}\right]_0^{20\ \mu s} + \frac{1}{L}[1.2\tau\ \text{nV}]_{20\ \mu s}^{30\ \mu s} = 0 + \frac{1}{16\ \mu\text{H}}\cdot 1\ \frac{\text{V}}{\text{s}^2}\cdot\left((20\ \mu s)^3 - 0\right) +$$

$$+ \frac{1}{16\ \mu\text{H}}\cdot(1.2\ \text{nV})\cdot(30\ \mu s - 20\ \mu s) = 1.250\ \text{nA}$$

---

## Problem 4.17

### *Solution:*

**Known quantities:**
Resistance value $R = 7\ \Omega$; inductance value $L = 7\ \text{mH}$ ; capacitance value $C = 0.5\ \mu\text{F}$ ; the voltage across the components as shown in Figure P4.17.

**Find:**
The current through each component.

**Assumptions:**
$i_R(t \le 0) = i_L(t \le 0) = i_C(t \le 0) = 0$

**Analysis:**
Since the voltage waveform is piecewise continuous, integration and differentiation can only be performed over each continuous segment. Where not indicated, *t* is expressed in seconds.

For $t \le 0$:

$$i_R(t) = i_L(t) = i_C(t) = 0$$

For $0 < t < 5\ \text{ms}$:

$$v(t) = m_v t + q_v$$

where:

$$m_v = \frac{[15\ \text{V}] - [0]}{[5\ \text{ms}] - [0]} = 3\ \frac{\text{V}}{\text{ms}}$$

$$q_{v_C} = 0\ \text{V}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{m_v \cdot t}{R} = \frac{3\ \frac{\text{V}}{\text{ms}}\cdot t}{7\ \Omega} = 428.6 \cdot t\ \text{A}$$

$$i_L(t) = \frac{1}{L}\int_0^t v(\tau)d\tau = \frac{1}{L}\int_0^t m_v \cdot \tau\, d\tau = \frac{1}{L}\left[\frac{1}{2}m_v \cdot \tau^2\right]_0^t = \frac{1}{2L}m_v \cdot t^2 =$$

$$= \frac{1}{2\cdot 7\ \text{mH}}\cdot 3\ \frac{\text{V}}{\text{ms}}\cdot t^2 = 214.3\cdot 10^3 \cdot t^2\ \text{A}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[m_v t] = Cm_v = \left(0.5\ \frac{\mu\text{As}}{\text{V}}\right)\left(3\ \frac{\text{V}}{\text{ms}}\right) = 1.5\ \text{mA}$$

For $t = 5$ ms:

$$i_R(5\cdot 10^{-3}) = 428.6\cdot t\ \text{A} = 428.6\cdot 5\cdot 10^{-3}\ \text{A} = 2.143\ \text{A}$$

$$i_L(5\cdot 10^{-3}) = 214.3\cdot 10^3\cdot t^2\ \text{A} = 214.3\cdot 10^3\cdot(5\cdot 10^{-3})^2\ \text{A} = 5.357\ \text{A}$$

$$i_C(5\cdot 10^{-3}) = 1.5\ \text{mA}$$

For $5\ \text{ms} < t < 10\ \text{ms}$:

$$v(t) = c_v = 15\ \text{V}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{c_v}{R} = \frac{15\ \text{V}}{7\ \Omega} = 2.143\ \text{A}$$

$$i_L(t) = i_L(5\cdot 10^{-3}) + \frac{1}{L}\int_{5\cdot 10^{-3}}^{t} v(\tau)d\tau = i_L(5\cdot 10^{-3}) + \frac{1}{L}\int_{5\cdot 10^{-3}}^{t} c_v\, d\tau =$$

$$= i_L(5\cdot 10^{-3}) + \frac{1}{L}[c_v\cdot\tau]_{5\cdot 10^{-3}}^{t} = i_L(5\cdot 10^{-3}) + \frac{1}{L}\cdot c_v\cdot(t - 5\cdot 10^{-3}) =$$

$$= 5.357\ \text{A} + \frac{1}{7\ \text{mH}}\cdot 15\ \text{V}\cdot(t - 5\cdot 10^{-3}\ \text{s}) = -5.357 + 2.143\cdot 10^3\cdot t\ \text{A}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[c_v] = 0$$

For $t = 10$ ms:

$$i_R(0.01) = 2.143\ \text{A}$$

$$i_L(0.01) = -5.357 + 2.143\cdot 10^3\cdot t\ \text{A} = -5.357 + 2.143\cdot 10^3\cdot 0.01\ \text{A} = 16.07\ \text{A}$$

$$i_C(0.01) = 0$$

For $t > 10$ ms:

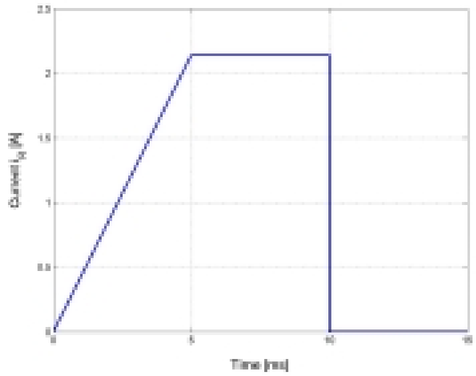
$$v(t) = 0$$

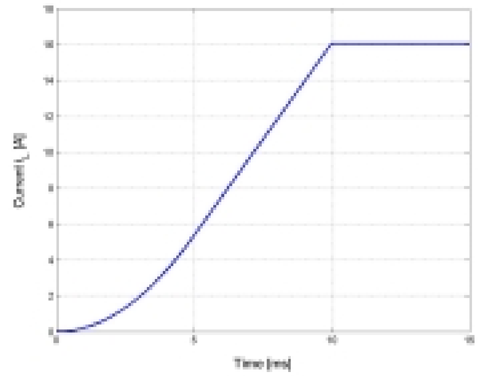
$$i_R(t) = \frac{v(t)}{R} = \frac{0}{R} = 0$$

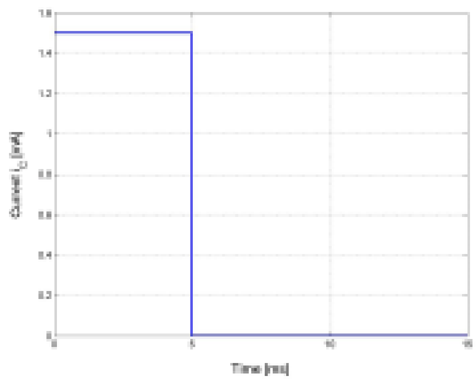
$$i_L(t) = i_L(10\cdot 10^{-3}) + \frac{1}{L}\int_{0\cdot 10^{-3}}^{t} v(\tau)d\tau = i_L(10\cdot 10^{-3}) + \frac{1}{L}\int_{0\cdot 10^{-3}}^{t} 0\, d\tau =$$

$$= i_L(10\cdot 10^{-3}) + \frac{1}{L}[0]_{10\cdot 10^{-3}}^{t} = i_L(10\cdot 10^{-3}) = 16.07\ \text{A}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d}{dt}[0] = 0$$







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### Problem 4.18

## *Solution:*

**Known quantities:**
The voltage across and the current through an ideal capacitor as shown in Figure P4.18.

**Find:**
The capacitance of the capacitor.

**Analysis:**
Considering the period: $-2.5\ \mu\text{s} < t < +2.5\ \mu\text{s}$ :

$$i_c = C\frac{dv_c}{dt} = C\frac{\Delta v_c}{\Delta t}$$, since the voltage has a linear waveform.

Substituting:

$$12\ \text{A} = C\frac{[+10\ \text{V}]-[-10\ \text{V}]}{5\ \mu\text{s}} \Rightarrow C = 12\ \text{A}\cdot\frac{5}{20}\frac{\mu\text{s}}{\text{V}} = 3\ \mu\text{F}$$

---

### Problem 4.19

## *Solution:*

**Known quantities:**
The voltage across and the current through an ideal inductor as shown in Figure P4.19.

**Find:**
The inductance of the inductor.

**Analysis:**

$$v_L = L\frac{di_L}{dt} = L\frac{\Delta i_L}{\Delta t}$$ , since the current has a linear waveform.

Substituting:

$$2\ \text{V} = L\frac{[2\ \text{A}]-[1\ \text{A}]}{10\ \text{ms}-5\ \text{ms}} \Rightarrow L = 2\ \text{V}\cdot\frac{5}{1}\frac{\text{ms}}{\text{A}} = 10\ \text{mH}$$

---

## Problem 4.20

### *Solution:*

**Known quantities:**
The voltage across and the current through an ideal capacitor as shown in Figure P4.20.

**Find:**
The capacitance of the capacitor.

**Analysis:**
Considering the period: $0 < t < 5\ \text{ms}$ :

$$i_c = C\frac{dv_c}{dt} = C\frac{\Delta v_c}{\Delta t}$$ , since the voltage has a linear waveform.

Substituting:

$$1.5\ \text{mA} = C\frac{[15\ \text{V}]-[0]}{5\ \text{ms}} \Rightarrow C = 1.5\ \text{mA}\cdot\frac{5}{15}\frac{\text{ms}}{\text{V}} = 0.5\ \mu\text{F}$$

---

## Problem 4.21

### *Solution:*

**Known quantities:**
The voltage across and the current through an ideal capacitor as shown in Figure P4.21.

**Find:**
The capacitance of the capacitor.

**Analysis:**
Considering the period: $0 < t < 5\ \text{ms}$ :

$$i_c = C\frac{dv_c}{dt} = C\frac{\Delta v_c}{\Delta t}$$ , since the voltage has a linear waveform.

Substituting:

$$3\ \text{mA} = C\frac{[7\ \text{V}]-[0]}{5\ \text{ms}} \Rightarrow C = 3\ \text{mA}\cdot\frac{5}{7}\frac{\text{ms}}{\text{V}} = 2.14\ \mu\text{F}$$

---

## *Section 4.2: Time-Dependent Signals*

### Problem 4.22

## *Solution:*

**Known quantities:**
The signal $x(t) = 2\cos(\omega t) + 2.5$.

**Find:**
The average and rms value of the signal.

**Analysis:**
The average value is:

$$\langle v(t) \rangle = \frac{\varpi}{2\pi}\left[\int_0^{\frac{2\pi}{\omega}} 2\cos(\omega t)dt + \int_0^{\frac{2\pi}{\omega}} (2.5)dt\right] = \frac{1}{2\pi}\left[-\sin(\varpi t)\Big|_0^{\frac{2\pi}{\omega}} + 2.5t\Big|_0^{\frac{2\pi}{\omega}}\right]$$

$$= \frac{1}{2\pi}[\sin(0) - \sin(2\pi)] + 2.5 = \frac{1}{2\pi}[0-0] + 2.5 = 2.5$$

The rms value is:

$$x_{rms} = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} (2\cos(\omega t) + 2.5)^2 \, dt} = \sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \left[4\cdot[\cos(\omega t)]^2 + 10\cdot\cos(\omega t) + 6.25\right]dt} =$$

$$= \sqrt{\frac{\omega}{2\pi}\cdot\left[4\cdot\int_0^{\frac{2\pi}{\omega}}[\cos(\omega t)]^2 dt + 10\cdot\int_0^{\frac{2\pi}{\omega}}\sin(\omega t)dt + 6.25\cdot\int_0^{\frac{2\pi}{\omega}} dt\right]} =$$

$$= \sqrt{\frac{\omega}{2\pi}\cdot\left[4\cdot\frac{1}{2}\cdot\frac{2\pi}{\omega} + 10\cdot 0 + 6.25\cdot\frac{2\pi}{\omega}\right]} = \sqrt{8.25} = 2.87$$

Note: The integral of a sinusoid over an integer number of period is identically zero. This is a useful and important result.

---

### Problem 4.23

## *Solution:*

**Known quantities:**
The sinusoidal voltage $v(t)$ of 110 V rms shown in Figure P4.23.

**Find:**
The average and rms voltage.

**Analysis:**
The rms value of a sinusoidal is equal to 0.707 times the peak value:

$$V_{peak} = 110\sqrt{2}$$

The average value is:

$$\langle v(t)\rangle = \frac{1}{2\pi}\left[\int_0^{\theta} 110\sqrt{2}\sin(t)dt + \int_{2\pi-\theta}^{2\pi} 110\sqrt{2}\sin(t)dt\right] =$$

$$= \frac{1}{2\pi}\left[-110\sqrt{2}\cos(t)\Big|_0^{\theta} - 110\sqrt{2}\cos(t)\Big|_{2\pi-\theta}^{2\pi}\right] =$$

$$= -\frac{50\sqrt{2}}{\pi}\left[\cos(\theta) - 1 + \cos(2\pi) - \cos(2\pi-\theta)\right] = 0$$

The rms value is:

$$v_{rms} = \left\{\frac{1}{2\pi}\left(\int_0^{\theta}\left(110\sqrt{2}\sin(t)\right)^2 dt + \int_{2\pi-\theta}^{2\pi}\left(110\sqrt{2}\sin(t)\right)^2 dt\right)\right\}^{1/2} =$$

$$= \left\{\frac{12100}{\pi}\left(\frac{1}{2}\left(-\cos(t)\sin(t)+t\right)\Big|_0^{\theta} + \frac{1}{2}\left(-\cos(t)\sin(t)+t\right)\Big|_{2\pi-\theta}^{2\pi}\right)\right\}^{1/2} =$$

$$= \left\{\frac{6050}{\pi}\left(-\cos(\theta)\sin(\theta)+\theta+2\pi-\left(-\cos(2\pi-\theta)\sin(2\pi-\theta)+2\pi-\theta\right)\right)\right\}^{1/2} =$$

$$= \left\{\frac{6050}{\pi}(2\theta)\right\}^{1/2} = 110\sqrt{\frac{\theta}{\pi}}$$

---

## Problem 4.24

### *Solution:*

**Known quantities:**

The sinusoidal voltage $v(t)$ of 110 V rms shown in Figure P4.23.

**Find:**

The angle $\theta$ that correspond to delivering exactly one-half of the total available power in the waveform to a resistive load.

**Analysis:**

From $v_{rms} = 110\sqrt{\frac{\theta}{\pi}}$, we obtain:

$$v_{rms}^2 = 110^2\frac{\theta}{\pi} = \frac{110^2}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{2}$$

---

## Problem 4.25

### *Solution:*

**Known quantities:**

The signal $v(t)$ shown in Figure P4.25.

**Find:**

The ratio between average and rms value of the signal.

**Analysis:**
The average value is:

$$\langle v\rangle = \frac{1}{0.004}\left[\int_0^{0.002}(-9)dt + \int_{0.002}^{0.004}(1)dt\right] = 250(-0.018+0.002) = -4 \text{ V}$$

The rms value is:

8

$$v_{rms} = \sqrt{\frac{1}{0.004}\left[\int_0^{0.002}(-9)^2 dt + \int_{0.002}^{0.004}(1)^2 dt\right]} = \sqrt{250\cdot[81\cdot 0.002 + 0.004 - 0.002]} = 6.40 \text{ V}$$

Therefor,

$$\frac{\langle v\rangle}{v_{rms}} = -\frac{4}{6.40} = -0.625$$

---

### Problem 4.26

## *Solution:*

**Known quantities:**
The signal $i(t)$ shown in Figure P4.26.

**Find:**
The power dissipated by a 1-Ω resistor.

**Analysis:**
The rms value is:

$$i_{rms} = \sqrt{\frac{1}{p}\int_0^p \left(10\cdot sin^2(t)\right)^2 dt} = \sqrt{\frac{1}{p}\int_0^p 100\cdot sin^4(t)dt} = \sqrt{\frac{1}{p}\cdot 100\cdot\frac{3p}{8}} = 6.12 \text{ A}$$

Therefore, the power dissipated by a 1-Ω resistor is:

$$P_{1\Omega} = Ri_{rms}^2 = (1)(6.12)^2 \text{ W} = 37.5 \text{ W}$$

---

### Problem 4.27

## *Solution:*

**Known quantities:**
The signal $x(t)$ shown in Figure P4.27.

**Find:**
The average and rms value of the signal.

**Analysis:**
The average value is:

$$\langle V\rangle = \frac{1}{T}\int_{t_0}^{t_0+\tau} V_m dt = \frac{t}{T}V_m$$

where $t_0$ is the left-hand side of the pulse.

The rms value is:

$$V_{rms} = \sqrt{\frac{1}{T}\int_{0}^{t_0+\tau} V_m^2 dt} = \sqrt{\frac{t}{T}}V_m$$

Therefore,

$$\frac{\langle V \rangle}{V_{rms}} = \sqrt{\frac{t}{T}}$$

---

### Problem 4.28

## *Solution:*

**Known quantities:**
The signal $i(t)$ shown in Figure P4.28.

**Find:**
The rms value of the signal.

**Analysis:**
The rms value is:

$$i_{rms} = \sqrt{\frac{1}{T}\left[\int_{0}^{\frac{T}{4}}\left(\frac{8}{T}t\right)^2 dt + \int_{\frac{T}{4}}^{\frac{3T}{4}}\left(-\frac{8}{T}t+4\right)^2 dt + \int_{\frac{3T}{4}}^{T}\left(\frac{8}{T}t-8\right)^2 dt\right]} =$$

$$= \sqrt{\frac{1}{T}\left[\int_{0}^{\frac{T}{4}}\left(\frac{64}{T^2}t^2\right)dt + \int_{\frac{T}{4}}^{\frac{3T}{4}}\left(\frac{64}{T^2}t^2 - \frac{64}{T}t+16\right)^2 dt + \int_{\frac{3T}{4}}^{T}\left(\frac{64}{T^2}t^2 - \frac{128}{T}t+64\right)^2 dt\right]} =$$

$$= \sqrt{\frac{1}{T}\left[\frac{1}{3}T + 9T - 18T + 12T - \frac{1}{3}T + 2T - 4T + \frac{64}{3}T - 64T + 64T - 9T + 36T - 48T\right]} =$$

$$= \sqrt{\frac{1}{T}\left[\frac{4}{3}T\right]} = \frac{2}{\sqrt{3}} = 1.15 \text{ A}$$

---

### Problem 4.29

## *Solution:*

**Known quantities:**
The signal $v(t)$.

**Find:**
The rms value of the signal.

**Analysis:**
The rms value is:

$$v_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi}(v(t))^2 d(\omega t)}$$

$$v^2{}_{rms} = \frac{1}{2\pi}\int_0^{2\pi}\left(V_{DC} + V_0\, cos(\omega t)\right)^2 d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi}\left(V^2{}_{DC} + 2V_{DC}V_0\, cos(\omega t) + V_0^2\, cos^2(\omega t)\right)d(\omega t) =$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(V_{DC}^2 + 2V_{DC}V_0\, cos(\omega t) + \frac{V_0^2}{2} + \frac{V_0^2}{2} cos^2(\omega t)\right)d(\omega t) =$$

$$= \frac{1}{2\pi}\left(V_{DC}^2\left[\omega t\right]_0^{2\pi} + 0 + \frac{V_0^2}{2}\left[\omega t\right]_0^{2\pi} + 0\right) = \frac{1}{2\pi}\left(V_{DC}^2(2\pi - 0) + 0 + \frac{V_0^2}{2}(2\pi - 0) + 0\right)$$

$$v_{rms} = \sqrt{V_{DC}^2 + \frac{V_0^2}{2}} = \sqrt{(50\ \text{V})^2 + \frac{1}{2}(70.7\ \text{V})^2} = 70.7\ \text{V}$$

Note:

1. T = period in units of time and $\omega t$ = period in angular units, i.e., $2\pi$ radians. Considering $\omega t$ as a single variable is useful when dealing with sinusoids.
2. A sinusoid integrated over one or more whole periods gives 0 which is very useful.

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## *Section 4.4: Phasor Solution of Circuits with Sinusoidal Excitation*

| **Focus on Methodology: Phasors** |
|---|

1. Any sinusoidal signal may be mathematically represented in one of two ways: a **time domain form**: $v(t) \quad A\cos( \quad t \quad )$, and a frequency domain form: $\mathbf{V}(j \quad ) \quad Ae^{j} \quad A$ . Note the $j\omega$ in the notation $\mathbf{V}(j \quad )$, indicating the $e^{j \quad t}$ dependence of the phasor. In the remainder of this chapter, bold uppercase quantities indicate phasor voltages and currents.
2. A phasor is a complex number, expressed in polar form, consisting of a *magnitude* equal to the peak amplitude of the sinusoidal signal and a *phase angle* equalto the phase shift of the sinusoidal signal *referenced to a cosine signal*.
3. when one is using phaso notation, it is important tonote the specific frequency w of the sinusoidal signal,since this is not explicitly apparent in the phasor expression.

### Problem 4.30

## *Solution:*

**Known quantities:**
The current through and the voltage across a component.

**Find:**
a) Whether the component is a resistor, capacitor, inductor
b) The value of the component in ohms, farads, or henrys.

**Analysis:**
a) The current and the voltage can be expressed in phasor form:

$$\mathrm{I} = 17\angle -15^o \ \mathrm{mA}, \quad \mathrm{V} = 3.5\angle 75^o \ \mathrm{V}$$

$$Z = \frac{\mathrm{V}}{\mathrm{I}} = \frac{3.5\angle 75^o \ \mathrm{V}}{17\angle -15^o \ \mathrm{mA}} = 205.9\angle 90^o \ \Omega = 0 + j \cdot 205.9\,\Omega$$

The impedance has a positive imaginary or reactive component and a positive angle of 90 degree indicating that this is an inductor (see Fig. 4.39).

b) $$Z_L = j \cdot X_L = j \cdot \omega L = j \cdot 205.9\,\Omega \Rightarrow L = \frac{205.9\,\Omega}{628.3 \ \frac{\mathrm{rad}}{\mathrm{s}}} = 327.7 \ \mathrm{m}\frac{\mathrm{Vs}}{\mathrm{A}} = 327.7 \ \mathrm{mH}$$

---

### Problem 4.31

## *Solution:*

**Known quantities:**
The waveform of a signal shown in Figure P4.31.

**Find:**
The sinusoidal description of the signal.

**Analysis:**
From the graph of Figure P4.31:

$$\phi = +\frac{\pi}{3}\frac{180^o}{\pi} = 60^o,\ V_0 = 170\ V,\ \omega = 2\pi f = \frac{2\pi}{T}$$ … not given.

$$v_r(t) = V_0\ cos(\omega t + \phi) = 170\,cos(\omega t + 60^o)\ \text{V}$$

Phasor form:

$$\text{V} = V_0\angle\phi = 170\angle 60^o\ \ \text{V} = 170\ \text{V}\cdot e^{j60^o}$$

---

### Problem 4.32

## *Solution:*

**Known quantities:**
The waveform of a signal shown in Figure P4.32.

**Find:**
The sinusoidal description of the signal.

**Analysis:**
From graph:

$$\phi = -\frac{3\pi}{4}\frac{180^o}{\pi} = -135^o,\ I_0 = 8\ \text{mA},\ \omega = 2\pi f = \frac{2\pi}{T} = 1571\ \frac{\text{rad}}{\text{s}}$$

$$i(t) = I_0\ cos(\omega t + \phi) = 8\,cos\left(1571\ \frac{\text{rad}}{\text{s}}\cdot t - 135^o\right)\ \text{mA}$$

Phasor form:

$$\text{I} = I_0\angle\phi = 8\angle -135^o\ \ \text{mA} = 8\ \text{mA}\cdot e^{-j135^o}$$

---

### Problem 4.33

## *Solution:*

**Known quantities:**
The waveform of a signal shown in Figure P4.33.

**Find:**
The sinusoidal description of the signal.

**Analysis:**
From graph:

$$\phi = -\frac{3\pi}{4}\frac{180^o}{\pi} = -135^o,\ I_0 = 8\ \text{mA},\ \omega = 2\pi f = \frac{2\pi}{T} = 1571\ \frac{\text{rad}}{\text{s}}$$

$$i(t) = I_0\ cos(\omega t + \phi) = 8\,cos\left(1571\ \frac{\text{rad}}{\text{s}}\cdot t - 135^o\right)\ \text{mA}$$

Phasor form:

$$\text{I} = I_0\angle\phi = 8\angle -135^o\ \ \text{mA} = 8\ \text{mA}\cdot e^{-j135^o}$$

---

## Problem 4.34

### *Solution:*

**Known quantities:**

The current through $i(t) = I_0 \cos(\omega t + 45^o)$, $I_0 = 3$ **mA**, $\omega = 6.283 \ \frac{\textbf{rad}}{\textbf{s}}$, and the voltage across $v(t) = V_0 \cos(\omega t)$, $V_0 = 700$ **mV**, $\omega = 6.283 \ \frac{\textbf{rad}}{\textbf{s}}$ an electrical component.

**Find:**

a) Whether the component is inductive or capacitive.
b) The waveform of the instantaneous power $p(t)$ as a function of $\omega t$ over the range $0 < \omega t < 2\pi$.
c) The average power dissipated as heat in the component.
d) The same as b. and c. with the phase of the current equal to zero.

**Analysis:**

a) Phasor notation:

$$\mathbf{I} = 3\angle 45^o \ \textbf{mA}, \ \text{V} = 700\angle 0^o \ \text{mV}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{700\angle 0^o \ \textbf{mV}}{3\angle 90^o \ \textbf{mA}} = 233.3\angle -45^o \ \mathbf{\Omega} = 165.0 - j165.0 \ \mathbf{\Omega}$$

The component is inductive because it is lagging.

b)

$$p(t) = v(t)i(t) = V_0 I_0 \cos(\omega t + 45^o)\cos(\omega t) = \frac{1}{2}V_0 I_0 \left(\cos(2\omega t + 45^o) + \cos(45^o)\right) =$$

$$= \frac{1}{2}(700 \ \textbf{mV})(3 \ \textbf{mA})\left(\cos(2\omega t + 45^o) + 0.707\right) = \left(1050\cos(2\omega t + 45^o) + 742.4\right)\mu\mathbf{W}$$

c)

$$P = \frac{1}{\omega T}\int_0^{\omega T} p(t)d\omega t = \frac{1}{\omega T}\int_0^{\omega T}\left(1050 \ \mu\mathbf{W}\cos(2\omega t + 45^o) + 742.4\right)d\omega t =$$

$$= \frac{1}{\omega T}(1050 \ \mu\mathbf{W})\frac{1}{2}\left[\sin(2\omega t + 45^o)\right]_0^{2\pi} + \frac{1}{\omega T}742.4(\omega t)\Big|_0^{2\pi} =$$

$$= \frac{1}{\omega T}(1050 \ \mu\mathbf{W})\frac{1}{2}\left[\sin(765^o) - \sin(45°)\right] + 742.4 =$$

$$= \frac{1}{2\pi}(1050 \ \mu\mathbf{W})\frac{1}{2}[0 - 0] + 742.4 = 742.4\mu\mathbf{W}$$

d)

$$p(t) = v(t)i(t) = \frac{1}{2}V_0 I_0 \cos(\omega t)\cos(\omega t) = \frac{1}{2}V_0 I_0 \left(\cos(2\omega t) + \cos(0^o)\right) =$$

$$= \frac{1}{2}(700 \ \textbf{mV})(3 \ \textbf{mA})\left(\cos(2\omega t) + 1\right) = 1050\left(\cos(2\omega t) + 1\right) \ \mu\mathbf{W}$$

$$P = \frac{1}{\omega T}\int_0^{\omega t} p(t)d\omega t = \frac{1}{\omega T}\int_0^{\omega t}(1050\ \mu\mathbf{W}(\cos(2\omega t)+1))d\omega t =$$

$$= \frac{1}{\omega T}(1050\ \mu\mathbf{W})\left(\frac{1}{2}[\sin(2\omega t)]_0^{2\pi} + [\omega t]_0^{2\pi}\right) =$$

$$= \frac{1}{2\pi}(1050\ \mu\mathbf{W})\left(\frac{1}{2}(0-0)+(2\pi-0)\right) = 1050\ \mu\mathbf{W}$$

---

### Problem 4.35

## *Solution:*

**Known quantities:**
The values of the impedance, $R_1 = 2.3\ \text{k}\Omega$, $R_2 = 1.1\ \text{k}\Omega$, $L = 190\ \text{mH}$, $C = 55\ \text{nF}$ and the voltage applied to the circuit shown in Figure P4.35, $v_s(t) = 7\,cos(3000t + 30^o)\ \text{V}$.

**Find:**
The equivalent impedance of the circuit.

**Analysis:**

$$X_L = \omega L = \left(3\ \text{k}\frac{\text{rad}}{\text{s}}\right)(190\ \text{mH}) = 0.57\ \text{k}\Omega \Rightarrow Z_L = +j\cdot X_L = +j\cdot 0.57\ \text{k}\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3\ \text{k}\frac{\text{rad}}{\text{s}}\right)(55\ \text{nF})} = 6.061\ \text{k}\Omega \Rightarrow Z_C = -j\cdot X_C = -j\cdot 6.061\ \text{k}\Omega$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 2.3 + j\cdot 0.57\ \text{k}\Omega = 2.37\angle 13.92^o\ \text{k}\Omega$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 1.1 - j\cdot 6.061\ \text{k}\Omega = 6.16\angle -79.71^o\ \text{k}\Omega$$

$$Z_{eq} = \frac{Z_{eq1}\cdot Z_{eq2}}{Z_{eq1}+Z_{eq2}} = \frac{(2.37\angle 13.92^o\ \mathbf{k\Omega})\cdot(6.16\angle -79.71^o\ \mathbf{k\Omega})}{(2.3+j\cdot 0.57\ \mathbf{k\Omega})+(1.1-j\cdot 6.061\ \mathbf{k\Omega})} =$$

$$= \frac{14.60\angle -65.79^o\ \mathbf{k\Omega^2}}{3.4 - j\cdot 5.491\ \mathbf{k\Omega}} = \frac{14.60\angle -65.79^o\ \mathbf{k\Omega^2}}{6.458\angle -58.23^o\ \mathbf{k\Omega}} = 2.261\angle -7.56^o\ \mathbf{k\Omega}$$

---

### Problem 4.36

## *Solution:*

**Known quantities:**
The values of the impedance, $R_1 = 3.3\ \text{k}\Omega$, $R_2 = 22\ \text{k}\Omega$, $L = 1.90\ \text{H}$, $C = 6.8\ \text{nF}$ and the voltage applied to the circuit shown in Figure P4.35, $v_s(t) = 636\,cos(3000t + 15^o)\ \text{V}$.

**Find:**
The equivalent impedance of the circuit.

**Analysis:**

$$X_L = \omega L = \left(3 \text{ k}\frac{\text{rad}}{\text{s}}\right)(1.90 \text{ H}) = 5.7 \text{ k}\Omega \Rightarrow Z_L = +j \cdot X_L = +j \cdot 5.7 \text{ k}\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(3 \text{ k}\frac{\text{rad}}{\text{s}}\right)(6.8 \text{ nF})} = 49.02 \text{ k}\Omega \Rightarrow Z_C = -j \cdot X_C = -j \cdot 49.02 \text{ k}\Omega$$

$$Z_{eq1} = Z_{R1} + Z_L = R_1 + jX_L = 3.3 + j \cdot 5.7 \text{ k}\Omega = 6.59\angle 59.93^o \text{ k}\Omega$$

$$Z_{eq2} = Z_{R1} + Z_C = R_1 - jX_C = 22 - j \cdot 49.02 \text{ k}\Omega = 53.73\angle -65.83^o \text{ k}\Omega$$

$$Z_{eq} = \frac{Z_{eq1} \cdot Z_{eq2}}{Z_{eq1} + Z_{eq2}} = \frac{(6.59\angle 59.93^o \textbf{ k}\Omega) \cdot (53.73\angle -65.83^o \textbf{ k}\Omega)}{(3.3 + j \cdot 5.7 \textbf{ k}\Omega) + (22 - j \cdot 49.02 \textbf{ k}\Omega)} =$$

$$= \frac{354.08\angle -5.9^o \textbf{ k}\Omega^2}{25.3 - j \cdot 43.32 \textbf{ k}\Omega} = \frac{354.08\angle -5.9^o \textbf{ k}\Omega^2}{50.17\angle -59.71^o \textbf{ k}\Omega} = 7.05\angle 53.81^o \textbf{ k}\Omega$$

---

## Problem 4.37

### *Solution:*

**Known quantities:**

The current in the circuit, $i_s(t) = I_0 \, cos(\omega t + 30^o)$, $I_0 = 13$ mA, $\omega = 1000 \frac{\text{rad}}{\text{s}}$, and the value of the capacitance present in the circuit shown in Figure P4.37 $C = 0.5 \ \mu\text{F}$ .

**Find:**

a) The phasor notation for the source current.
b) The impedance of the capacitor.
c) The voltage across the capacitor, showing all the passages and using phasor notation only.

**Analysis:**

a) Phasor notation:

$$\text{I}_s = I_0 \angle\phi = 13\angle 30^o \text{ mA}$$

b)

$$Z_C = -jX_C - j\frac{1}{\omega C} = -j\frac{1}{\left(1000 \frac{\text{rad}}{\text{s}}\right)(0.5 \ \mu\text{F})} = 0 - j2 \text{ k}\Omega = 2\angle -90^o \text{ k}\Omega$$

c)

$$\text{V}_C = \text{I}_s \cdot Z_C = (13\angle 30^o \text{ mA}) \cdot (2\angle -90^o \text{ k}\Omega) = 26\angle -60^o \text{ V}$$

$$v_C(t) = 26\, cos(1000t - 60^o) \text{ V}$$

Note that conversion from phasor notation to time notation or vice versa can be done at any time.

---

## Problem 4.38

### *Solution:*

**Known quantities:**
The values of two currents in the circuit shown in Figure P4.38: $i_1(t) = 141.4\,cos(\omega t + 135^o)$ mA ,
$i_2(t) = 50\,cos(\omega t + 53.13^o)$ mA , $\omega = 377\ \dfrac{\text{rad}}{\text{s}}$.

**Find:**
The current $i_3(t)$.

**Analysis:**
A solution using trigonometric identities is possible but inefficient, cumbersome, and takes a lot of time. Phasors are better! Note that one current is described with a sine and the other with a cosine function. When using phasors, all currents and voltages must be described with either sine functions or cosine functions. Which does not matter, but it is a good idea to adopt one and use it consistently. Therefore, first converts to cosines.

KCL: $-i_1(t) + i_2(t) + i_3(t) = 0 \ \Rightarrow\ +i_3(t) = i_1(t) - i_2(t)$

$$i_3(t) = 141.4\,cos(\omega t + 135^o)\text{ mA} - 50\,sin(\omega t - 53.13^o)\text{ mA} =$$
$$= 141.4\,cos(\omega t + 135^o)\text{ mA} - 50\,cos(\omega t - 53.13^o - 90^o)\text{ mA}$$

$$\text{I}_3 = 141.4\text{ mA }\angle 135^o - 50\text{ mA }\angle -143.13^o =$$
$$= (-99.98 + j\cdot 99.98)\text{ mA} - (-40.00 - j\cdot 30.00)\text{ mA} =$$
$$= (-59.98 + j\cdot 129.98)\text{ mA} = 143.2\text{ mA }\angle 114.8^o$$

$$i_3(t) = 143.2\,cos(\omega t + 114.8^o)\text{ mA}$$

If sine functions were used, the result in phasor notation would differ in phase by 90 degrees.

---

## Problem 4.39

### *Solution:*

**Known quantities:**
The values of the impedance, $Z_1 = 5.9\angle 7^o\ \text{k}\Omega$, $Z_2 = 2.3\angle 0^o\ \Omega$, $Z_3 = 17\angle 11^o\ \Omega$ and the voltages applied to the circuit shown in Figure P4.39, $v_{s1}(t) = v_{s2}(t) = 170\,cos(377t)$ V .

**Find:**
The current through $Z_3$.

**Analysis:**

$$\text{V}_{s1} = \text{V}_{s2} = 170\angle 0^o\text{ V} = (170 + j0)\text{ V}$$

KVL: $-\mathbf{V}_{s1} - \mathbf{V}_{s2} + \mathbf{I}_3 Z_3 = 0$

$$\text{I}_3 = \frac{\text{V}_{s1} + \text{V}_{s2}}{Z_3} = \frac{170\angle 0^o\text{ V} + 170\angle 0^o\text{ V}}{17\angle 11^o\ \Omega} = \frac{340\angle 0^o\text{ V}}{17\angle 11^o\ \Omega} = 20\angle -11^o\text{ A}$$

$$i_3(t) = 20\,cos\left(377\ \frac{\text{rad}}{\text{s}} \cdot t - 11^o\right)\ \text{A}$$

Note also:

KVL: $-\mathbf{V}_{s1} + \mathbf{I}_1 Z_1 = 0 \Rightarrow \mathbf{I}_1 = \frac{\mathbf{V}_{s1}}{Z_1}$, $-\mathbf{V}_{s2} - \mathbf{I}_2 Z_2 = 0 \Rightarrow \mathbf{I}_2 = -\frac{\mathbf{V}_{s2}}{Z_2}$

---

## Problem 4.40

### *Solution:*

**Known quantities:**

The values of the impedance in the circuit shown in Figure P4.40, $Z_s = (13000 + j\omega 3)\,\Omega$, $R = 120\,\Omega$, $L = 19\text{ mH}$, $C = 220\text{ pF}$.

**Find:**

The frequency such that the current $\mathbf{I}_i$ and the voltage $\mathbf{V}_0$ are in phase.

**Analysis:**

$Z_s$ is not a factor in this solution. Only R, L, and C will determine if the voltage across this combination is in phase with the current through it. If the voltage and current are in phase, then, the equivalent impedance must have an "imaginary" or reactive part which is zero!

$$Z_{eq} = \frac{\mathbf{V}_0}{\mathbf{I}_i} = |Z_{eq}|\angle 0^o = R_{eq} + jX_{eq},\ X_{eq}(\omega) = 0$$

$$Z_{eq} = \frac{(Z_R + Z_L)\cdot Z_C}{Z_R + Z_L + Z_C} = \frac{(R + jX_L)\cdot(-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jRX_C}{R + j(X_L - X_C)}\frac{R - j(X_L - X_C)}{R - j(X_L - X_C)} =$$

$$= \frac{[X_L X_C R - RX_C(X_L - X_C)] - j[R^2 X_C + X_L X_C (X_L - X_C)]}{R^2 + (X_L - X_C)^2}$$

At the resonant frequency the reactive component of this impedance must equal zero:

$$X_{eq}(\omega) = \frac{R^2 X_C + X_L X_C(X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0 \Rightarrow R^2 + X_L(X_L - X_C) = 0$$

$$R^2 + \omega L\left(\omega L - \frac{1}{\omega C}\right) = 0 \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(19\text{ mH})(220\text{ pF})} - \frac{(120\,\Omega)^2}{(19\text{ mH})^2}} =$$

$$= \sqrt{293.3\text{ G}\frac{\text{rad}}{\text{s}} - 39.89\text{ M}\frac{\text{rad}}{\text{s}}} = 489.1\text{ k}\frac{\text{rad}}{\text{s}}$$

Notes:

1. To separate the equivalent impedance into real (resistive) and "imaginary" (reactive) components, the denominator had to be "rationalized". This was done by multiplying numerator and denominator by the complex conjugate of the denominator, and multiplying term by term. Remember that $j^2 = -1$, etc.
2. The term with *R* had a negligible effect on the resonant frequency in this case. If *R* is sufficiently large, however, it will significantly affect the answer.

---

## Problem 4.41

### *Solution:*

**Known quantities:**
The circuit shown in Figure P4.40

**Find:**
The frequency such that the current $\mathbf{I}_i$ and the voltage $\mathbf{V}_0$ are in phase.

**Analysis:**
If the voltage and current are in phase, then, the equivalent impedance must have an "imaginary" or reactive part which is zero!

$$Z_{eq} = \frac{\mathbf{V}_0}{\mathbf{I}_i} = |Z_{eq}|\angle 0^o = R_{eq} + jX_{eq},\ X_{eq}(\omega) = 0$$

$$Z_{eq} = \frac{(Z_R + Z_L)\cdot Z_C}{Z_R + Z_L + Z_C} = \frac{(R + jX_L)\cdot(-jX_C)}{R + jX_L - jX_C} = \frac{X_L X_C - jRX_C}{R + j(X_L - X_C)} \frac{R - j(X_L - X_C)}{R - j(X_L - X_C)} =$$

$$= \frac{[X_L X_C R - RX_C(X_L - X_C)] - j[R^2 X_C + X_L X_C (X_L - X_C)]}{R^2 + (X_L - X_C)^2}$$

At the resonant frequency the reactive component of this impedance must equal zero:

$$X_{eq}(\omega) = \frac{R^2 X_C + X_L X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0 \Rightarrow R^2 + X_L(X_L - X_C) = 0$$

$$R^2 + \omega L\left(\omega L - \frac{1}{\omega C}\right) = 0 \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

---

## Problem 4.42

### *Solution:*

**Known quantities:**
The values of the impedance, $R_s = 50\ \Omega$, $R_c = 40\ \Omega$, $L = 20\ \mu\text{H}$, $C = 1.25\ \text{nF}$, and the voltage applied to the circuit shown in Figure P4.42, $v_s(t) = V_0\, cos(\omega t + 0^o)$, $V_0 = 10\ \text{V}$, $\omega = 6\ \text{M}\frac{\text{rad}}{\text{s}}$.

**Find:**
The current supplied by the source.

**Analysis:**
Assume clockwise currents:

$$X_L = \omega L = \left(6\ \text{M}\frac{\text{rad}}{\text{s}}\right)(20\ \mu\text{H}) = 1203\ \Omega \Rightarrow Z_L = 0 + j120\ \Omega = 120\angle 90^o\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(6\ \text{M}\frac{\text{rad}}{\text{s}}\right)(1.25\ \text{nF})} = 133.3\ \Omega \Rightarrow Z_C = 0 - j133.3\ \Omega = 133.3\angle -90^o\ \Omega$$

$$Z_{R_c} = 40 - j\,\Omega = 40\angle 0^o\ \Omega,\ Z_{R_s} = 50 - j\,\Omega = 50\angle 0^o\ \Omega$$

Equivalent impedances:

$$Z_{eq1} = Z_{R_c} + Z_L = 40 + j120\,\Omega = 126.5\angle 71.56^o\ \Omega$$

$$Z_{eq} = Z_{R_s} + \frac{Z_C \cdot Z_{eq1}}{Z_C + Z_{eq1}} = 50 + j0\,\Omega + \frac{(133.3\angle -90^o\ \Omega)\cdot(126.5\angle 71.56^o\ \Omega)}{133.3\angle -90^o\ \Omega + 126.5\angle 71.56^o\ \Omega} =$$

$$= 50 + j0\,\Omega + \frac{16.87\angle -18.44^o\ \text{k}\Omega^2}{42.161\angle -18.44^o\ \Omega} = 50\angle 0^o\ \Omega + 400\angle 0^o\ \Omega = 450\angle 0^o\ \Omega$$

OL: $$\text{I}_s = \frac{\text{V}_s}{Z_{eq}} = \frac{10\angle 0^o\ \text{V}}{450\angle 0^o\ \Omega} = 22.22\angle 0^o\ \text{mA} \Rightarrow i_s(t) = 22.22\,cos(\omega t + 0^o)\,\text{mA}$$

Note:
The equivalent impedance of the parallel combination is purely resistive; therefore, the frequency given is the resonant frequency of this network.

---

## Problem 4.43

### *Solution:*

**Known quantities:**
The values of the impedance and the voltage applied to the circuit shown in Figure P4.43.

**Find:**
The current in the circuit.

**Analysis:**
Assume clockwise currents:

$$\omega = 3\ \frac{\text{rad}}{\text{s}},\ V_S = 12\angle 0^o\ \text{V}$$

$$Z_C = \frac{1}{j\omega C} = -j\ \Omega,\ Z_L = j\omega L = j9\ \Omega \Rightarrow Z_{total} = 3 + j9 - j = 3 + j8\ \Omega$$

$$I = \frac{12}{3 + j8} = 0.4932 - j1.3151\ \mathbf{A} = 1.4045\angle -69.44^o\ \mathbf{A},$$

$$i(t) = 1.4\cos(\omega t - 69.4^o)\ \mathbf{A}$$

---

## Problem 4.44

### *Solution:*

**Known quantities:**
The values of the impedance and the current source shown in Figure P4.44.

**Find:**
The voltage.

**Analysis:**
Assume clockwise currents:

$$\omega = 2\ \frac{\text{rad}}{\text{s}},\ \text{I}_S = 10\angle 0^o\ \text{A},\ Z_L = j\omega L = j6\ \Omega,\ Z_C = \frac{1}{j\omega C} = -j1.5\Omega$$

$$Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{Z_L} + \frac{1}{Z_C}} = \frac{1}{\frac{1}{3} - j\frac{1}{6} + j\frac{2}{3}} = \frac{1}{0.33 + j0.5} = 0.9231 - j1.3846\ \Omega$$

$$V = I_S Z_{eq} = 10\text{ A} \cdot (0.9231 - j1.3846)\,\Omega = 9.231 - j13.846\ a*10\text{ V} = 16.641\angle -56.31^o\ \text{V}$$

---

### Problem 4.45

## *Solution:*

**Known quantities:**
The values of the impedance and the current source for the circuit shown in Figure P4.45.

**Find:**
The current $\mathbf{I}_1$.

**Analysis:**
Specifying the positive directions of the currents as in figure P4.45:

$$Z_{eq} = \frac{1}{\frac{1}{2} + \left(\frac{1}{-j4}\right)} = 1.79\angle 26.56^o\ \Omega$$

$$V_S = I_S Z_{eq} = (10\angle -22.5^o)\,\mathbf{A} \cdot (1.79\angle 26.56^o)\,\Omega = 17.9\angle 4.06^o\ \mathbf{V}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_S}{R} = 8.95\angle 4.06^o\ \mathbf{A}$$

---

### Problem 4.46

## *Solution:*

**Known quantities:**
The values of the impedance and the voltage source for circuit shown in Figure P4.46.

**Find:**
The voltage $\mathbf{V}_2$.

**Analysis:**
Specifying the positive directions as in figure P4.46:

$$Z_L = j\omega L = j12\ \Omega$$

$$V_2 = \frac{R_{6\Omega}}{R_{12\Omega} + Z_L + R_{6\Omega}} V = \frac{6\ \Omega}{(12 + j12 + 6)\,\Omega} 25\angle 0^o\ \mathbf{V} = \frac{150\angle 0^o\ \Omega}{18 + j12\ \Omega}\mathbf{V} = 6.93\angle -33.7^o\ \mathbf{V}$$

---

### Problem 4.47

## *Solution:*

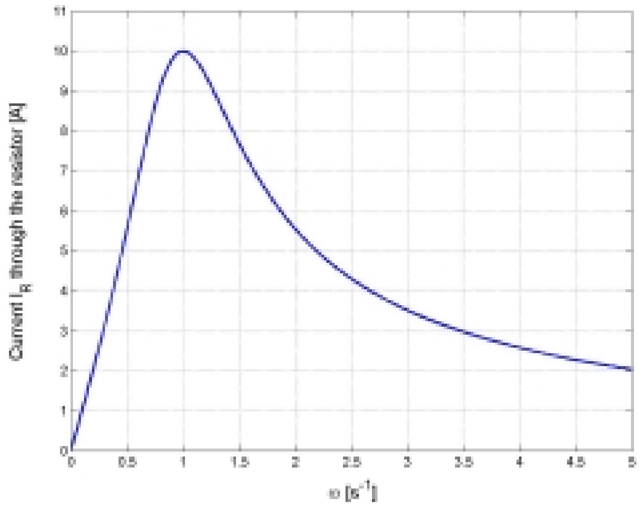
**Known quantities:**
The values of the impedance and the current source of circuit shown in Figure P4.44.

**Find:**
The value of $\omega$ for which the current through the resistor is maximum.

**Analysis:**
Assume clockwise currents:

$$\mathrm{I}_S = 10\angle 0^o \text{ A},\ Z_L = j\omega L = j3\omega\,\Omega,\ Z_C = \frac{1}{j\omega C} = -j\frac{3}{\omega}\Omega$$

$$I_R = \frac{\frac{1}{R}}{\frac{1}{R}+\frac{1}{Z_L}+\frac{1}{Z_C}} I_S = \frac{\frac{1}{3}}{\frac{1}{3}-j\frac{1}{3\omega}+j\frac{\omega}{3}} 10\angle 0^o = \frac{10\omega}{\omega + j(\omega^2-1)}$$




The maximum of $I_R$ is obtained for $\omega = 1.$ , therefore $i_R(t) = 10\cos(t)$.

---

## Problem 4.48

### *Solution:*

**Known quantities:**
The values of the impedance and the current source for circuit shown in Figure P4.48.

**Find:**
The current through the resistor.

**Analysis:**
Specifying the positive directions as in figure P4.48:
By current division:

$$\mathrm{I}_R = -\frac{\frac{1}{R}}{\frac{1}{R}+\frac{1}{Z_C}}\cdot \mathrm{I}_s = -\frac{1}{1+\frac{R}{j\omega C}}\cdot \mathrm{I}_s = -\frac{1}{1+j\omega RC}\cdot \mathrm{I}_s = -\frac{1-j\omega RC}{1+(\omega RC)^2}\cdot \mathrm{I}_s =$$

$$= -(0.0247 - j0.1552)\text{ A}\cdot 1\angle 0^o \text{ A} = 157\cdot 10^{-3}\angle 99.04^o \text{ A}$$

$$i_R(t) = 157\,cos(200\pi t + 99.04^o)\text{ mA}$$

---

## Problem 4.49

### *Solution:*

**Known quantities:**

The values of the reactance $X_L = 1\ \text{k}\Omega$, $X_C = 10\ \text{k}\Omega$, and the current source $\mathbf{I} = 10\angle 45^o\ \mathbf{mA}$ for circuit shown in Figure P4.49.

**Find:**

The voltage $v_{out}$.

**Analysis:**

Specifying the positive directions of the currents as in figure P4.49:

$$\mathbf{V}_{out} = Z_{eq}\mathbf{I} = (Z_L + Z_C)\mathbf{I} = (0 + jX_L + 0 - jX_C)\mathbf{I} = (j1\ \mathbf{k\Omega} - j10\ \mathbf{k\Omega})\cdot 10\angle 45^o\ \mathbf{mA} =$$

$$= (-j9\ \mathbf{k\Omega})\cdot 10\angle 45^o\ \mathbf{mA} = 9\angle -90^o\ \mathbf{k\Omega}\cdot 10\angle 45^o\ \mathbf{mA} = 90\angle -45^o\ \mathbf{V}$$

$$v_{out} = 90\cos(\omega t - 45^\circ)\ \mathbf{V}$$

---

## Problem 4.50

### *Solution:*

**Known quantities:**

The circuit shown in Figure P4.50, the values of the resistance, $R = 2\ \Omega$, capacitance, $C = 1/8\ \text{F}$, inductance, $L = 1/4\ \text{H}$, and the frequency $\omega = 4\ \frac{\text{rad}}{\text{s}}$.

**Find:**

The impedance $Z$.

**Analysis:**

$$Z_L = j\omega L = j4\frac{1}{4}\ \Omega = j\ \Omega,\ Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} = -j\frac{1}{4\cdot(1/8)} = -j2\ \Omega$$

$$Z = Z_L + Z_C \| R = Z_L + \frac{1}{\frac{1}{Z_C} + \frac{1}{R}} = j + \frac{1}{\frac{1}{-j2} + \frac{1}{2}} = j + \frac{j2}{-1+j} = j + \frac{(j2)}{(-1+j)}\frac{(-1-j)}{(-1-j)}$$

$$= j + \frac{j2(-1-j)}{1+1} = j - j + 1 = 1\ \Omega$$

---

## Problem 4.51

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.51, the values of the resistance, $R = 3\ \Omega$, capacitance, $C = 1/10\ \text{F}$, inductance, $L = 4/5\ \text{H}$, and the frequency $\omega = 5\ \frac{\text{rad}}{\text{s}}$.

**Find:**
The admittance $Y$ .
**Analysis:**

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j5\cdot(1/10)} = -j2\,\Omega,\; Z_L = j5\cdot\frac{4}{5} = j4\,\Omega$$

$$Y = \frac{1}{Z} = \frac{1}{Z_C \| (R+Z_L)} = \frac{1}{\dfrac{1}{\dfrac{1}{Z_C}+\dfrac{1}{R+Z_L}}} = \frac{1}{Z_C} + \frac{1}{R+Z_L} = \frac{1}{-j2} + \frac{1}{3+j4}$$

$$= j\frac{1}{2} + \frac{(1)}{(3+j4)}\frac{(3-j4)}{(3-j4)} = j\frac{1}{2} + \frac{3-j4}{9+16}$$

$$= j\frac{1}{2} + \frac{3-j4}{25} = j\frac{1}{2} + \frac{3}{25} - j\frac{4}{25} = 0.12 + j0.34\ \mathbf{S}$$

---

## *Section 4.5: AC Circuit Analysis methods*

**Focus on Methodology: AC Circuit Analysis**

1. Identify the sinusoidal source(s) and note the excitation frequency.
2. Convert the source(s) tophasor form.
3. Represent each circuit element by ots impedance.
4. Splve the resulting phasor circuit, using appropriate circuit analysis tools.
5. Convert the (phasor-form) answer to its time-domain equivalent, using equation 4.50.

### Problem 4.52

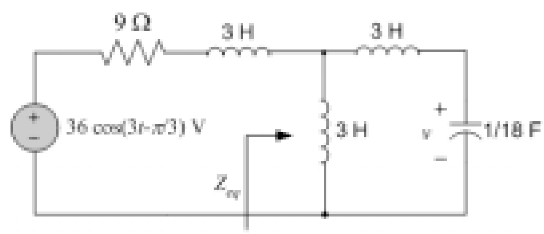
### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.52, the values of the resistance, $R = 9\ \Omega$, capacitance, $C = 1/18\ \text{F}$, inductance, $L_1 = 3\ \text{H}$, $L_2 = 3\ \text{H}$, $L_3 = 3\ \text{H}$, and the voltage source $v_s(t) = 36 cos\left(3t - \frac{\pi}{3}\right)\ \text{V}$.

**Find:**

The voltage across the capacitance $v$ using phasor tehcniques.




**Analysis:**

$$\omega = 3\ \frac{\text{rad}}{\text{s}},\ V_s = 36\angle -60°\ \text{V}$$

$$Z_{L_2} = j\omega L_2 = j3\cdot 3 = j9\ \Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j3\cdot(1/18)} = -j6\ \Omega$$

$$Z_{L_3} = j\omega L_3 = j3\cdot 3 = j9\ \Omega$$

$$Z_{eq} = \frac{1}{Z_{L_3} \| (Z_{L_2} + Z_C)} = \frac{1}{\frac{1}{Z_{L_3}} + \frac{1}{(Z_{L_2} + Z_C)}} = \frac{1}{\frac{1}{j9} + \frac{1}{j3}} = \frac{j9}{4} = 2.25\angle 90^o\ \Omega$$

$$Z_T = Z_R + Z_{L_1} + Z_{eq} = 9 + j3\cdot 3 + j2.25 = 9 + j11.25 = 14.407\angle 51.34^o\ \Omega$$

$$I = \frac{V_S}{Z_T} = \frac{36\angle -60^o\ \text{V}}{14.407\angle 51.34^0\ \Omega} = 2.499\angle -111.34^o\ \text{A}$$

$$V_{eq} = IZ_{eq} = (2.499\angle -111.34^o)(2.25\angle 90^o) = 5.623\angle -21.34^o\ \text{V}$$

$$V = \frac{Z_C}{(Z_{L_2} + Z_C)} V_{eq} = \frac{-j6}{j3} 5.623\angle -21.34^o = 11.25\angle 158.66^o\ \text{V}$$

$$v = 11.25\cos(3t - 158.66^o)\ \text{V}$$

---

## Problem 4.53

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.53, the values of the resistance, $R = 5\ \Omega$, capacitance, $C = 1/2\ \text{F}$, inductance, $L_1 = 0.5\ \text{H}$, $L_2 = 1\ \text{H}$, $L_2 = 10\ \text{H}$, and the current source $i_s(t) = 6\,cos(2t)\ \text{A}$.

**Find:**

The current through the inductance $i_{L_2}$.

**Analysis:**

$$\omega = 2\ \frac{\text{rad}}{\text{s}},\ Z_{L_2} = j\omega L_2 = j2\,\Omega$$

$$Z_C = \frac{1}{j\omega C} = -j\,\Omega,\ Z_{L_3} = j\omega L_3 = j20\ \Omega$$

$$I = \frac{Z_{L_3} + Z_C}{\left(Z_{L_3} + Z_C\right) + \left(R + Z_{L_2}\right)} I_S = \frac{j20 - j}{(j20 - j) + (5 + j2)} 6\angle 0^o$$

$$= \frac{j19}{5 + j21} 6\angle 0^o = 5.28\angle 13.4^o\ \textbf{A}$$

$$i = 5.28\cos\left(2t + 13.4^o\right)\ \textbf{A}$$

---

## Problem 4.54

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.54 the values of the resistance, $R_S = R_L = 500\,\Omega$, $R = 1\ k\Omega$ and the inductance, $L = 10\ \text{mH}$.

**Find:**

a) The Thèvenin equivalent circuit if the voltage applied to the circuit is $v_S(t) = 10\,cos(1{,}000t)$.

b) The Thèvenin equivalent circuit if the voltage applied to the circuit is $v_S(t) = 10\,cos(1{,}000{,}000t)$.

**Analysis:**

a) $Z_L = j\omega L = j1000\ \frac{\text{rad}}{\text{s}} \cdot 10\ \text{mH} = j10\,\Omega,$

The equivalent impedance is:

$$Z_T = \frac{Z_L \cdot R}{(Z_L + R)} + R_S = \frac{(j10)1000}{j10 + 1000} + 500 = 500 + \frac{j10^3}{100 + j} = 500.1 + j9.999\ \Omega$$

The equivalent Thèvenin voltage is:

$$\text{V}_T = \text{V}_S = 10\angle 0^o\ \text{V}$$

b) $Z_L = j\omega L = j10^6\ \frac{\text{rad}}{\text{s}} \cdot 10\ \text{mH} = j10^4\ \Omega,$

The equivalent impedance is:

$$Z_T = \frac{Z_L \cdot R}{(Z_L + R)} + R_S = \frac{(j10^4)1000}{j10^4 + 1000} + 500 = 500 + \frac{j10^4}{1 + j10} = 1490.1 + j99.01\ \Omega$$

The equivalent Thèvenin voltage is:

$$\mathrm{V}_T = \mathrm{V}_S = 10\angle 0^o\ \mathrm{V}$$

---

### Problem 4.55

## *Solution:*

Circuit shown in Figure P4.55 the values of the impedance, $L = 0.1\ \mathrm{H}$, capacitance, $C = 100\ \mu\mathrm{F}$ , and the voltage source $v_{in}(t) = 12\cos(10t)\ \mathrm{V}$.

**Find:**

The Thèvenin equivalent of the circuit as seen by the load resistor $R_L$.

**Analysis:**

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j10\ \frac{\mathrm{rad}}{\mathrm{s}} \cdot 100\ \mu\mathrm{F}} = -j1000\ \Omega$$

$$Z_L = j\omega L = j10\ \frac{\mathrm{rad}}{\mathrm{s}} \cdot 0.1\ \mathrm{H} = j1\ \Omega$$

The equivalent impedance is:

$$Z_T = Z_L \| Z_C = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{j(-j1000)}{j - j1000} = \frac{1000}{-j999} = 1.001\angle 90^o\ \Omega = j1.001\ \Omega$$

The Thèvenin voltage is:

$$V_T = \frac{Z_C}{Z_L + Z_C} V_{in} = \frac{-j1000}{j - j1000} \cdot 12\angle 0^o = \frac{1000}{999} \cdot 12\angle 0^o = 12.012\angle 0^o\ \mathrm{V}$$

---

### Problem 4.56

## *Solution:*

**Known quantities:**

Circuit shown in Figure P4.56 the values of the resistance, $R_1 = 4\ \Omega$, $R_2 = 4\ \Omega$, capacitance, $C = 1/4\ \mathrm{F}$, inductance, $L = 2\ \mathrm{H}$, and the voltage source $v_s(t) = 2\,cos(2t)\ \mathrm{V}$.

**Find:**

The current in the circuit $i_L(t)$ using phasor techniques.

**Analysis:**

$$\mathrm{V}_S(t) = 2\angle 0^o\ \mathrm{V}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\ \frac{1}{4}} = -j2\ \Omega$$

$$Z_L = j\omega L = j2 \cdot 2 = j4\ \Omega$$

Applying the voltage divider rule:

$$V_L = \frac{(Z_L \parallel (Z_C + Z_2))}{Z_1 + (Z_L \parallel (Z_C + Z_2))} V_S = \frac{4\angle 36.8°}{4\angle 0° + 4\angle 36.8°} 2\angle 0^o = 1.05\angle 18.4° \text{ V}$$

Therefore, the current is:

$$I_L = \frac{V_L}{Z_L} = \frac{1.05\angle 18.4°}{4\angle 90°} = 0.2635\angle -71.6^o \text{ A}$$

$$i_L(t) = 0.2635\cos(2t - 71.6^o) \text{ A}$$

---

## Problem 4.57

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.57, the values of the resistance, $R_1 = 75\ \Omega$, $R_2 = 100\ \Omega$, capacitance, $C = 1\ \mu\text{F}$ , inductance, $L = 0.5\ \text{H}$, and the voltage source $v_s(t) = 15\,cos(1{,}500t)$ V .

**Find:**

The currents in the circuit $i_1(t)$ and $i_2(t)$.

**Analysis:**

In the phasor domain:

$$Z_C = \frac{-j}{1500(1\times 10^{-6})} = -j\frac{2000}{3} = -j666.7\ \Omega,\ Z_L = j(1500)(0.5) = j750\ \Omega$$

By applying KVL in the first loop, we have

$$V_S = R_1 I_1 + Z_C(I_1 - I_2)$$

By applying KVL in the second loop, we have

$$0 = (Z_C)(I_2 - I_1) + (Z_L + R_2)I_2$$

That is:

$$\begin{cases} 15\angle 0^o = \left(75 - j\dfrac{2000}{3}\right)I_1 + j\dfrac{2000}{3}I_2 \\ 0 = j\dfrac{2000}{3}I_1 + \left(100 + j\dfrac{250}{3}\right)I_2 \end{cases}$$

By solving above equations, we have

$$I_1 = 3.8\cdot 10^{-3}\angle 46.6^o \text{ A}$$

$$I_2 = 19.6\cdot 10^{-3}\angle -83.2^o \text{ A}$$

$$i_1(t) = 3.8\cos(1{,}500t + 46.6^o) \text{ mA}$$

$$i_2(t) = 19.6\cos(1{,}500t - 83.2^o) \text{ mA}$$

---

## Problem 4.58

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.58, the values of the resistance, $R_1 = 40\ \Omega$, $R_2 = 10\ \Omega$, capacitance, $C = 500\ \mu\text{F}$ , inductance, $L = 0.2\ \text{H}$, and the current source $i_s(t) = 40\cos(100t)$ A.

**Find:**
The voltages in the circuit $v_1(t)$ and $v_2(t)$.

**Analysis:**

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{100 \cdot 500 \cdot 10^{-6}} = -j20\,\Omega\ ,\ Z_L = j\omega L = j100 \cdot 0.2 = j20\,\Omega$$

Applying KCL at node 1, we have:

$$I_S = \frac{V_1}{R_1} + \frac{V_1 - V_2}{Z_C} \Rightarrow I_S = \left(\frac{1}{R_1} + \frac{1}{Z_C}\right)V_1 - \frac{1}{Z_C}V_2 \Rightarrow 40\angle 0^o = \left(\frac{1}{40} + \frac{j}{20}\right)V_1 - \frac{j}{20}V_2$$

Applying KCL at node 2, we have

$$\frac{V_1 - V_2}{Z_C} = \frac{V_2}{R_2} + \frac{V_2}{Z_L} \Rightarrow \frac{V_1}{Z_C} = \left(\frac{1}{R_2} + \frac{1}{Z_L} + \frac{1}{Z_C}\right)V_2 \Rightarrow j\frac{V_1}{20} = \left(\frac{1}{10} - j\frac{1}{20} + j\frac{1}{20}\right)V_2$$

Therefore:

$$\begin{cases} 40\angle 0^o = \left(\frac{1}{40} + \frac{j}{20}\right)V_1 - \frac{j}{20}V_2 \\ j\frac{V_1}{20} = \left(\frac{1}{10}\right)V_2 \end{cases} \Rightarrow \begin{cases} 40\angle 0^o = \left(\frac{1}{40} + \frac{j}{20}\right)(-j2V_2) - \frac{j}{20}V_2 \\ V_1 = -j2V_2 \end{cases} \Rightarrow$$

$$\begin{cases} 40\angle 0^o = -\frac{j}{20}V_2 + \frac{1}{10}V_2 - \frac{j}{20}V_2 = \left(\frac{1}{10} - \frac{j}{10}\right)V_2 \\ V_1 = -j2V_2 \end{cases}$$

$$V_2 = \frac{40\angle 0^o}{\left(\frac{1}{10} - \frac{j}{10}\right)} = 282.84\angle 45^o \text{ V},\ V_1 = -j2V_2 = 565.68\angle -45^o \text{ V}$$

$$v_2(t) = 282.84\cos(100t + 45^o) \text{ V},\ v_1(t) = 568.68\cos(100t - 45^o) \text{ V}$$

---

## Problem 4.59

### *Solution:*

**Known quantities:**
The circuit called Wheatstone bridge shown in Figure P4.59.

a) The balanced status for the bridge: $v_{ab} = 0$.
b) The values of the resistance, $R_1 = 100\,\Omega$, $R_2 = 1\,\Omega$, the capacitance, $C_3 = 4.7\ \mu\text{F}$ , the inductance, $L_3 = 0.098\ \text{H}$ , that are necessary to balance the bridge: $v_{ab} = 0$ , and the voltage applied to the bridge, $v_s = 24\,sin(2{,}000t)\ \text{V}$ .

**Find:**

a) The unknown reactance $X_4$ in terms of the circuit elements.
b) The value of the unknown reactance $X_4$ .
c) The source frequency that should be avoided in this circuit.

**Analysis:**

a) Assuming a balanced circuit, we have $v_{ab} = 0$ , that is, $v_a = v_b$

From the voltage divider: $\dfrac{R_2}{jX_{L_3} - jX_{C_3} + R_2} = \dfrac{jX_4}{R_1 + jX_4} \Rightarrow \dfrac{R_2}{j\omega L_3 - \dfrac{j}{\omega C_3} + R_2} = \dfrac{jX_4}{R_1 + jX_4}$

Inverting both sides and equating imaginary parts:

$$R_1 R_2 = \left(-\omega L_3 + \frac{1}{\omega C_3}\right) X_4 \Rightarrow X_4 = \frac{R_1 R_2}{\left(\dfrac{1}{\omega C_3} - \omega L_3\right)}$$

b)

$$X_4 = \frac{100 \cdot 1}{\left(\dfrac{1}{2000 \cdot 4.7 \cdot 10^{-6}} - 2000 \cdot 0.098\right)} = -1.116\,\Omega$$

Negative reactance implies that the component is a capacitor.

$$\frac{1}{\omega C} = 1.116\,\Omega \Rightarrow C = \frac{1}{\omega \cdot 1.116} = 448\ \mu\text{F}$$

c)

If the reactances of $L_3$ and $C_3$ cancel, the bridge can not measure $X_4$. Thus, the condition to be avoided is:

$$\omega L_3 - \frac{1}{\omega C_3} = 0 \Rightarrow L_3 C_3 = \frac{1}{\omega^2} \Rightarrow \omega = \frac{1}{\sqrt{L_3 C_3}} = \frac{1}{\sqrt{0.098 \cdot 4.7 \cdot 10^{-6}}} = 1473\ \frac{\text{rad}}{\text{s}}$$

$$f = 234.5\ \text{Hz}$$

---

## Problem 4.60

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.56, the values of the resistance, $R_1 = 4\,\Omega$, $R_2 = 4\,\Omega$, capacitance, $C = 1/4\ \text{F}$, inductance, $L = 2\ \text{H}$, and the voltage source $v_s(t) = 2\cos(2t)\ \text{V}$.

**Find:**

The Thévenin impedance seen by resistor $R_2$.

**Analysis:**

$$Z_T = (R_1 \| Z_L) + (Z_C) = (4 \| j4) + (-j2) = j2(1-j) - j2 = 2 + j2 + (-j2) = 2\,\Omega$$

---

## Problem 4.61

### *Solution:*

**Known quantities:**

Circuit shown in Figure P4.58, the values of the resistance, $R_1 = 10\,\Omega$, $R_2 = 40\,\Omega$, capacitance, $C = 500\ \mu\text{F}$, inductance, $L = 0.2\ \text{H}$, and the current source $i_s(t) = 40\cos(100t)\ \text{A}$.

**Find:**

The Thévenin voltage seen by inductance $L$.

**Analysis:**

The Thévenin equivalent voltage source is the open-circuit voltage at the load terminals:

$$\mathbf{V}_T = R_2\mathbf{I}_2 = 40\mathbf{I}_2$$

From the current division, we have

$$\mathrm{I}_2 = \frac{R_1}{(R_2 + Z_C) + R_1}\mathrm{I}_S = \frac{10}{(40\text{-}j20) + 10} 40\angle 0^o = 7.43\angle 21.8^o \text{ A}$$

$$\mathrm{V}_T = R_2\mathrm{I}_2 = 40 \cdot 7.43\angle 21.8^o = 297\angle 21.8^o \text{ V}$$

$$v_T(t) = 297\,cos(100t + 21.8^o) \text{ V}$$

---

### Problem 4.62

## *Solution:*

**Known quantities:**

Circuit shown in Figure P4.62, the values of the impedance, $R = 8\,\Omega$, $Z_C = -j8\,\Omega$, $Z_L = j8\,\Omega$, and the voltage source $\mathrm{V}_s = 5\angle -30^o$ V.

**Find:**

The Thévenin equivalent circuit seen from the terminals a-b.

**Analysis:**

The Thévenin equivalent circuit is given by:

$$V_{TH} = \left(\frac{8 + j8}{8 + j8 - j8}\right)5\angle -30° = (1 + j)5\angle -30^o = 7.07\angle 15 \text{ V}$$

$$Z_{TH} = \frac{(8 + j8)(-j8)}{8 + j8 - j8} = (8 - j8) = 8\sqrt{2}\angle -45^o\ \Omega$$

---

### Problem 4.63

## *Solution:*

**Known quantities:**

Circuit shown in Figure P4.56, the values of the resistance, $R_1 = 4\,\Omega$, $R_2 = 4\,\Omega$, capacitance, $C = 1/4$ F, inductance, $L = 2$ H, and the voltage source $v_s(t) = 2\,cos(2t)$ V.

**Find:**

The Thévenin equivalent voltage seen by the resistor $R_2$.

**Analysis:**

The Thévenin equivalent circuit is given by:

$$\mathrm{V}_T = \frac{j4}{4 + j4} 2\angle 0^o = (1 + j) = \sqrt{2}\angle 45^o = 1.414\angle 45^o \text{ V}$$

$$v_T(t) = 1.414\,cos(2t + 45^o) \text{ V}$$

---

## Problem 4.64

### *Solution:*

**Known quantities:**
Circuit shown in Figure P4.56, the values of the resistance, $R_1 = 4\,\Omega$, $R_2 = 4\,\Omega$, capacitance, $C = 1/4$ F, inductance, $L = 2$ H, and the voltage source $v_s(t) = 2\cos(2t)$ V.

**Find:**
The Norton equivalent circuit seen by the resistor $R_2$.

**Analysis:**
From the result of Problem 4.60, we have $Z_T = 2\,\Omega$. From the current divider:

$$\mathbf{I}_N = \frac{j4}{j4\text{-}j2}\mathbf{I} = 2\mathbf{I}$$

and

$$j4 \| -j2 = \frac{(-j2)(j4)}{j2} = -j4$$

The current is:

$$\mathrm{I} = \frac{2\angle 0^o}{4\text{-}j4} = \frac{\sqrt{2}}{4}\angle 45^o = 0.353\angle 45^o \text{ A}$$

Therefore:

$$\mathrm{I}_N = 2\mathrm{I} = 0.707\angle 45^o \text{ A}$$

---

## Problem 4.65

### *Solution:*

**Known quantities:**
Circuit shown in Figure P4.66.

**Find:**
The equations required to solve for the loop currents in the circuit in:
a. Integral-differential form;
b. Phasor form.

**Analysis:**

KVL: $-v_S + i_1 R_S + v_c(0) + \frac{1}{C}\int_0^t (i_1 - i_2)dt + (i_1 - i_2)R_1 = 0$

KVL: $(i_2 - i_1)R_1 - v_c(0) + \frac{1}{C}\int_0^t (i_2 - i_1)dt + L\frac{di_2}{dt} + i_2 R_2 = 0$

Note: The initial voltage across the capacitor must, in general, be considered. It is modeled as an ideal voltage source in series with the capacitor.

KVL: $-\mathbf{V}_S + \mathbf{I}_1 R_S + (\mathbf{I}_1 - \mathbf{I}_2)Z_C + (\mathbf{I}_1 - \mathbf{I}_2)R_1 = 0$

KVL: $(\mathbf{I}_2 - \mathbf{I}_1)R_1 + (\mathbf{I}_2 - \mathbf{I}_1)Z_C + +\mathbf{I}_2 Z_L + \mathbf{I}_2 R_2 = 0$

Note:
1. The *i-v* characteristics of the inductor and capacitor, i.e. the integral and derivative, have been replaced here by the impedance.
2. This form of the equation is applicable only when the waveforms of the currents and voltages are sinusoids!

---

## Problem 4.66

### *Solution:*

**Known quantities:**
Circuit shown in Figure P4.65.

**Find:**
The node equations required to solve for all currents and voltages in the circuit.

**Analysis:**

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3$$
$$\mathbf{V}_S - \mathbf{I}_1 R_S = \mathbf{I}_2 (Z_C + R_1) + \mathbf{I}_3 (Z_L + R_2)$$

---

## Problem 4.67

### *Solution:*

**Known quantities:**
The voltages at the nodes of the circuit shown in Figure P4.67, $V_a = 450\angle 0^o$ V, $V_b = 440\angle 30^o$ V, $V_c = 420\angle -200^o$ V, $V_{bc} = 779.5\angle 5.621^o$ V, $V_{cd} = 153.9\angle 68.93^o$ V, $V_{ba} = 230.6\angle 107.4^o$ V, and the voltage sources, $v_{s1} = 450 cos(\omega t)$ V, $v_{s2} = 450 cos(\omega t)$ V.

**Find:**
The new values of $\mathbf{V}_b$ and $\mathbf{V}_{bc}$, if the ground is moved from Node e to Node d.

**Analysis:**
A node voltage is defined as the voltage between a node and the ground node. If the ground node is changed, then all node voltages in the circuit will change. With the ground at Node *d*:

$$V_b = V_{bd} = V_{be} + V_{ed} = V_{be} + V_{s2} = 440\angle 30^o \text{ V} + 450\angle 0^o \text{ V} =$$
$$= (381.1 + j220.0) \text{ V} + (450 + j0) \text{ V} = 831.1 + j220.0 \text{ V} = 859.6\angle 14.83^o \text{ V}$$

The voltages between any two nodes in a circuit do not depend on which is the ground node; therefore, the voltage between Node *b* and Node *c* remains the same when the ground is moved from Node *e* to Node *d*:

$$V_{bc} = 779.5\angle 5.621^o \text{ V}$$

---

## Problem 4.68

### *Solution:*

**Known quantities:**
Circuit shown in Figure P4.68, the values of the resistance, $R_L = 120\ \Omega$, the capacitance, $C = 12.5\ \mu\text{F}$, and the inductance, $L = 60$ mH, and the voltage source, $v_i = 4 cos(1{,}000t + 30^o)$ V.

**Find:**
The new value of $\mathbf{V}_0$.

**Analysis:**
The circuit has 3 unknown mesh currents but only 1 unknown node voltage.

$$Z_C = -jX_C = -j\frac{1}{\omega C} = -j\frac{1}{\left(1{,}000\ \text{k}\frac{\text{rad}}{\text{s}}\right)(12.5\ \mu\text{F})} = -j80\ \Omega = 80\angle -90^o\ \Omega$$

$$Z_L = jX_L = j\omega L = j\left(1{,}000\ \text{k}\frac{\text{rad}}{\text{s}}\right)(60\ \text{mH}) = j60\ \Omega = 60\angle 90^o\ \Omega$$

Reference phasor: $V_i = 4\angle 30^o\ \text{V}$

KCL: $\frac{\mathbf{V}_0 - 0}{Z_{R_L}} + \frac{\mathbf{V}_0 - 0}{Z_C} + \frac{\mathbf{V}_0 - \mathbf{V}_i}{Z_L} = 0$

$$\text{V}_0 = \frac{\frac{\text{V}_i}{Z_L}}{\frac{1}{Z_{R_L}} + \frac{1}{Z_C} + \frac{1}{Z_L}} = \frac{\text{V}_i}{\frac{Z_L}{Z_{R_L}} + \frac{Z_L}{Z_C} + 1} = \frac{4\angle 30^o\ \text{V}}{\frac{60\angle 90^o\ \Omega}{120\angle 0^o\ \Omega} + \frac{60\angle 90^o\ \Omega}{80\angle -90^o\ \Omega} + 1} =$$

$$= \frac{4\angle 30^o\ \text{V}}{0.5\angle 90^o + 0.75\angle 180^o + 1} = \frac{4\angle 30^o\ \text{V}}{(0 + j0.5) + (-0.75 + j0) + (1 + j0)} =$$

$$= \frac{4\angle 30^o\ \text{V}}{0.25 + j0.5} = \frac{4\angle 30^o\ \text{V}}{0.559\angle 63.43^o} = 7.155\angle -33.43^o\ \text{V}$$

$$v_0(t) = 7.155\,cos\left(\omega t - 33.43^o\right)\ \text{V}$$

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## Problem 4.69

### *Solution:*

**Known quantities:**
Circuit shown in Figure P4.69, the mesh currents and node voltages,
$i_1(t) = 3.127\,cos\left(\omega t - 47.28^o\right)\ \text{A}$, $i_2(t) = 3.914\,cos\left(\omega t - 102.0^o\right)\ \text{A}$,
$i_3(t) = 1.900\,cos\left(\omega t + 37.50^o\right)\ \text{A}$, $v_1(t) = 130.0\,cos\left(\omega t + 10.08^o\right)\ \text{V}$,
$v_2(t) = 130.0\,cos\left(\omega t - 25.00^o\right)\ \text{V}$, where $\omega = 377.0\ \frac{\text{rad}}{\text{s}}$.

**Find:**
One of the following: $L_1$, $C_2$, $R_3$, $L_3$.

**Analysis:**
KCL: $-\mathbf{I}_1 + \mathbf{I}_{Z_1} + \mathbf{I}_3 = 0$

$$\text{I}_{Z_1} = \text{I}_1 - \text{I}_3 = (2.121 - j2.297)\ \text{A} - (1.507 + j1.157)\ \text{A} = 3.508\angle -79.92^o\ \text{A}$$

OL: $Z_1 = \frac{\text{V}_1}{\text{I}_{Z_1}} = \frac{130\angle 10.08^o\ \text{V}}{3.508\angle -79.92^o\ \text{A}} = 37.05\angle 90^o\ \Omega = \omega L_1\angle 90^o$

$$L_1 = \frac{37.05\ \Omega}{377\ \frac{\text{rad}}{\text{s}}} = 98.29\ \text{mH}$$

KCL: $\mathbf{I}_2 + \mathbf{I}_{Z_2} - \mathbf{I}_3 = 0$

$$I_{Z_2} = I_3 - I_2 = (1.507 - j1.157)\ \text{A} - (-0.8138 - j3.828)\ \text{A} = 5.499\angle 65.03^o\ \text{A}$$

OL: $$Z_2 = \frac{V_2}{I_{Z_2}} = \frac{130\angle 24.97^o\ \text{V}}{5.499\angle 65.03^o\ \text{A}} = 23.64\angle -90^o\ \Omega = \frac{1}{\omega C_2}\angle -90^o$$

$$C_2 = \frac{1}{\left(377\ \frac{\text{rad}}{\text{s}}\right)(23.64\,\Omega)} = 112.2\ \mu\text{F}$$

KVL: $$\mathbf{V}_2 - \mathbf{V}_1 + \mathbf{V}_{Z_3} = 0$$

$$V_{Z_3} = V_1 - V_2 = (128.0 + j22.75)\ \text{V} - (117.8 - j54.88)\ \text{V} = 78.29\angle 82.56^o\ \text{V}$$

OL: $$Z_3 = \frac{V_{Z_3}}{I_3} = \frac{78.29\angle 82.56^o\ \text{V}}{1.9\angle 37.5^o\ \text{A}} = 4.21\angle 45.06^o\ \Omega = 29.11 + j29.17\ \Omega = R_3 + j\omega L_3$$

$$R_3 = 29.11\,\Omega,\ R_3 = \frac{29.17\ \Omega}{377\ \frac{\text{rad}}{\text{s}}} = 77.37\ \text{mH}$$

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